13th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2009, Hammamet, Tunisia, 16-21 October 2009

# EFFECTIVITY OF HYPERGEOMETRIC FUNCTION APPLICATION IN NUMERICAL SIMULATION OF HELICOPTER ROTOR BLADES THEORY

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## ABSTRACT
*Efficiency and justification of hypergeometric functions application in achieving simple formulas used in numerical simulation of helicopter rotor blades theory are presented in this paper.*
*First of all, basic equations of stream field over helicopter rotor are formulated, their decomposition is made and mean induced velocity harmonics are integrally presented. Theoretical basis of hypergeometric function application in transformation of integral equations of k - bladed rotor average induced velocity into special functions then follows. All necessary conditions for transformation hypergeometric functions into special functions are defined.*
*Various variants of integral transformation of expressions obtained in that way are presented here by numerical simulation and convenient solutions are found among them. This approach to effectivity of hypergeometric function application in helicopter rotor blades theory by numerical simulation allows achieving of synthetic method which can be used to define helicopter k - bladed main rotor optimal characteristics.*
**Keywords:** hypergeometric functions, unsteady flow, numerical simulation, k-blade, panel method, free vortex model, helicopter rotor aerodynamics

## 1. BASIC ASSUMPTIONS AND MUTUAL RELATIONS
In practical rotor calculations based on disc theory it can be assumed that circulation along supporting line is constant over blade azimuth angle. This assumption does not cause large differences in induced velocity computation at small values of $\mu$, and it significantly simplifies the computation.

This assumption is applied only for induced velocities calculation which represents the basis for further determination of actual values of angles of attack blade section and variable circulation $\overline{\Gamma}(\hat{\rho},\theta)$ which are necessary for rotor characteristics calculation.
Let the k-bladed rotor with diameter 2R and center in origin of Cartesian system Oxyz be placed in undisturbed flow field with velocity V. Rotor is rotating around y-axis with angular velocity $\omega$. Direction of velocity V forms with xz plane arbitrary angle $\alpha$. Rotor blade is presented by radial segment of supporting line with circulation varying with radius $\rho\left(0\le\rho\le R\right)$ and with constant circulation over azimuth angle $\theta$.

It is assumed that free vortex elements separating from supporting line are moving in space Oxyz along with particles of undisturbed flow field forming vortex shade in form of pitched spiral surface. Induced velocity V is calculated in arbitrary point of xz plane. That point is defined by polar coordinates: radius $r$ and azimuth angle $\psi$.

In order to simplify the calculation dimensions coefficients defined by following expressions will be used:

$$\overline{V} = \frac{V}{\omega R}; \quad \overline{\Gamma} = \frac{\Gamma}{\omega R^2}; \quad \overline{\rho} = \frac{\rho}{R}; \; \hat{\rho} = \frac{\rho}{r}; \quad \overline{r} = \frac{r}{R}$$

Induced velocity can be presented in following integral form [1]:

$$\overline{v} = \overline{v}_r + \frac{k}{4\pi\overline{V}} \int_0^1 \frac{\partial\overline{\Gamma}}{\partial\overline{\rho}} \Phi_p d\overline{\rho} + \frac{k}{4\pi\overline{r}} \int_0^1 \frac{\partial\overline{\Gamma}}{\partial\overline{\rho}} \Phi_q d\overline{\rho}$$

Therefore second term in expression (1) presents velocity field component symmetrical in regard to x-axis, and third term presents velocity field component asymmetrical in regard to x-axis. Let the velocity $\overline{v}$ be presented in form of Fourie series progression:

$$\overline{v} = \overline{v}_r + \sum_{n=1}^{\infty} \left( \overline{v}_{cn} \cos n\psi - \overline{v}_{sn} \sin n\psi \right)$$

and coefficients determined as

$$\overline{v}_{cn} = \frac{1}{\pi} \int_0^{2\pi} \overline{v} \cos n\psi \, d\psi;$$

$$\overline{v}_{sn} = \frac{1}{\pi} \int_0^{2\pi} \overline{v} \sin n\psi \, d\psi$$

by use of formula (1).

For nuclei calculation for large values of index $m$ it is convenient to use recurrence formula

$$S_{2m+3} = \frac{2m+1}{m+1} \left( 1 - 2\hat{\rho}^2 \right) S_{2m+1} - \frac{m}{m+1} S_{2m-1}$$

Graphic representation of function $S_{2m+1}$, for different $m$ are shown at Figure 1.

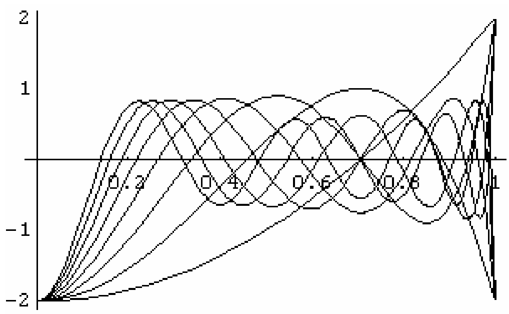


*Figure 1.*

Recurrence formula with other equations allows easy determination of nuclei $S_{2m}$ for every $m$ greater than zero. Transformations allows achieving of following integral representation of first and second order of Legendre function:

$$P_{m-\frac{1}{2}} \left( 1 - 2x^2 \right) = I_{2m}(x); \quad 0 \le x \langle 1;$$

$$Q_{m-\frac{1}{2}}\left(2x^2-1\right)=\left(-1\right)^m\frac{\pi}{2}I_{2m}\left(x\right);\quad x\rangle 1$$

where

$$I_{2m}=\frac{1}{\pi}\int_0^{\pi}T_{2m}\left(p\right)\frac{d\theta}{l};\quad \left(m=0,1,2,\ldots\right)$$

and $T_{2m}\left(p\right)$ is first order Chedyshev polynomial with $p=\frac{1}{l}\left(x\cos\theta-1\right)$ and $l=\sqrt{1+x^2-2x\cos\theta}$. When nuclei $S_n$ and coefficients $\overline{v}_{sn}$ are once determined it is possible to express nuclei $C_n$ for even indices $n=2m$ through Legendre polynomial

$$C_{2m}=\begin{cases}P_m\left(1-2\widehat{\rho}^2\right)-P_{m-1}\left(1-2\widehat{\rho}^2\right); & \widehat{\rho}\langle 1\\ 0; & \widehat{\rho}\rangle 1\end{cases}$$

Nuclei $C_{2m}$ can be expressed as hypergeometric functions by using assumptions:

$$C_{2m}=-2m\widehat{\rho}^2F\left(-m+1,m+1,2;\quad \widehat{\rho}^2\right)$$

For smaller values of index m elementary formulas are achieved:

$$\begin{aligned}C_2&=-2\widehat{\rho}^2;\\ C_4&=-4\widehat{\rho}^2+6\widehat{\rho}^2;\\ C_6&=-6\widehat{\rho}^2+24\widehat{\rho}^4-20\widehat{\rho}^6;\\ C_8&=-8\widehat{\rho}^2+60\widehat{\rho}^4-120\widehat{\rho}^6+70\widehat{\rho}^8\end{aligned}$$

Nuclei $C_n$ for odd indices $n=2m+1$ can be transformed into first and second order Jacobi functions which allows representation in form of elliptic integrals. For smaller $m$ follows:

$$C_1=\begin{cases}\frac{4}{\pi}\left[K\left(\widehat{\rho}\right)-E\left(\widehat{\rho}\right)\right]; & \widehat{\rho}\langle 1\\ \frac{4}{\pi}\widehat{\rho}\left[K\left(\frac{1}{\widehat{\rho}}\right)-E\left(\frac{1}{\widehat{\rho}}\right)\right]; & \widehat{\rho}\rangle 1\end{cases}$$

$$C_3=\begin{cases}\left(\frac{4}{3\pi}\left[\begin{aligned}&\left(1-4\widehat{\rho}^2\right)K\left(\widehat{\rho}\right)\\ &\quad-\left(1-8\widehat{\rho}^2\right)E\left(\widehat{\rho}\right)\end{aligned}\right]\right); & \widehat{\rho}\langle 1\\ \left(\frac{4}{3\pi}\widehat{\rho}\left[\begin{aligned}&\left(5-8\hat{\rho}^2\right)K\left(\frac{1}{\widehat{\rho}}\right)\\ &-\left(1-8\hat{\rho}^2\right)E\left(\frac{1}{\widehat{\rho}}\right)\end{aligned}\right]\right); & \widehat{\rho}\rangle 1\end{cases}$$

## 2. CONCLUSION

Efficiency of application of theory of hypergeometric function in rotor theory is reflected in achieving simple formulas representing velocity harmonics and representing basis for further numerical analyses of unsteady flow over helicopter rotor blades. Use of analytical presentation of induced velocity components significantly reduces working time and increases accuracy of calculation. Efficiency of applied method is shown by example and presented method shows great advantage in regard to classic approach to the problem solution because working time is shortened. Results achieved by this method are given by examples (Figure 2-6).

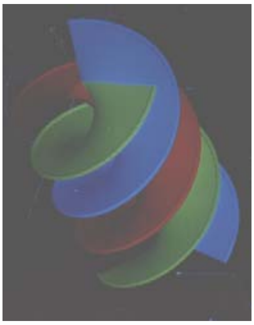

*Figure 2.*

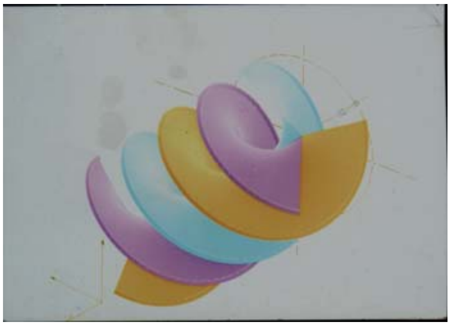

*Figure 3.*

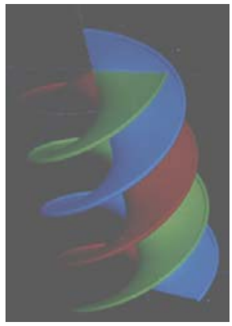

*Figure 4.*

*Figure 5.*

*Figure 6.*

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