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OBJECTIVE FUNCTION IN GENETIC ALGORITHM FOR MATERIAL BEHAVIOUR MODELING

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ABSTRACT
Constitutive modeling of material behaviour is becoming increasingly important in prediction of possible failures in highly loaded engineering components, and consequently, optimization of their design. Damage occurrence and accumulation within the material are described in this paper by means of models of kinematic and isotropic hardening according to Chaboche material model. Since the material model is non-linear, its parameter identification requires complex numerical procedures. Genetic algorithm is used for the determination of these parameters because of its capability to provide very good approximation of the solution in systems with large number of unknown variables. For the application of genetic algorithm to parameter identification, inverse analysis must be primarily defined. It is used as a tool to influence calculated stress-strain values with experimental ones. In order to choose proper objective function for inverse analysis among already existent and newly developed ones the research is performed to investigate it's influence on material behaviour modeling.
Keywords: material model, genetic algorithm, objective function

1. INTRODUCTION

Material behaviour modeling plays very important role in structural components design and it's fatigue analysis. Material models differ in the range of material properties they can describe and proportionally in complexity for their definition. Complex material models are characterized by numerous material parameters that have to be carefully identified to follow material behaviour as accurately as possible. Due to the complexity of chosen Chaboche's material model [1,2], it is neccessary to use complex numerical procedures to identify material parameters. The usage of evolutionary algorithms is proposed because of their advantageous characteristics, mainly considering insensitivity to errors in measured data, reliability to achieve convergence to accurate results, improbability for convergence to local minima and it's robustness considering the choice of objective function [3,4]. Genetic algorithm is stochastic search method for obtaining good approximate solutions in complex problems [5]. It is based on mechanisms of natural evolution and genetic principles. The genetic algorithm creates a population of solutions and applies genetic operators, such as scaling, selection, mutation and crossover to evolve the solutions in order to find the best ones. The proper evolution of population is assured by choosing adequate genetic operators in order to achieve fast convergence to global optima. One of the main premises in genetic algorithm application for parameter identification is the choice of objective function for inverse problem solution. There are numerous published papers that suggest different objective functions for the problem solution. In order to evaluate these suggestions and the influence of objective function on simulating material behaviour by parameter identification with genetic algorithm usage, the most common ones are investigated [6,7,8], and also their utterly modified versions that are proposed.

2. CONSTITUTIVE MATERIAL MODEL

2.1. Basic notions of constitutive theories

Since material behaviour is modeled considering it's cyclic loading and isothermal conditions, the small strain framework is assumed. The basic assumption is that strain tensor consists of two parts, an elastic strain tensor and plastic one:

$$\tilde{\varepsilon} = \tilde{\varepsilon}_{\mathrm{e}} + \tilde{\varepsilon}_{\mathrm{p}} . \tag{1}$$

Elastic strain tensor corresponds to Hooke's law of linear elasticity. Considering that the hardening induced by plastic flow can be described by combining kinematic and isotropic hardening, the elasticity domain can be given by the condition:

$$f = \left\| \tilde{\sigma} - \tilde{X} \right\| - R - k \leq 0 , \tag{2}$$

where k is initial yield surface size, $\tilde{X}$ is back stress, which represents kinematic hardening and R is the increase of yield surface size, which represents isotropic hardening. Von Mises domain of elasticity is given by:

$$f = \sqrt{3/2 \left(\tilde{\sigma}' - \tilde{X}'\right)\left(\tilde{\sigma}' - \tilde{X}'\right)} - R - k \leq 0 , \tag{3}$$

where $\tilde{\sigma}'$ and $\tilde{X}'$ are deviatoric tensor parts. The accumulated plastic strain rate follows normality rule:

$$\dot{\tilde{\varepsilon}}_{\mathrm{p}} = \dot{\lambda} \, \partial f / \partial \tilde{\sigma} , \tag{4}$$

where $\dot{\lambda}$ is plastic multiplier, derived from consistency condition $\dot{f} = 0$.

2.2. Constitutive equations

Strain hardening of material is described by isotropic and kinematic hardening rules. Isotropic hardening rule is defined by:

$$\mathrm{d}R = b\left(R_{\infty} - R\right)\mathrm{d}p , \tag{5}$$

where dp is accumulated plastic strain, R_{∞} is the boundary of isotropic hardening and b and R are parameters used to describe evolution of a yield surface. The exponential equation is used for determination of parameter b [1,2,5]. The non-linear kinematic hardening rule by Armstrong and Frederick [1,2,5]

$$\mathrm{d}X = \frac{2}{3} C \mathrm{d}\varepsilon_{\mathrm{i}}^{\mathrm{p}} - \gamma X \ \mathrm{d}p , \tag{6}$$

can be decomposed in three parts, as Chaboche proposed [1]:

$$X = \sum_{n=1}^{3} X^{(n)} , \tag{7}$$

where material parameters are: $X_{\infty}^{(1)}, X_{\infty}^{(2)}, X_{\infty}^{(3)}, \gamma^{(1)}, \gamma^{(2)}$ and $\gamma^{(3)}$. The non-linear hardening rule is described by the expression [1,2,5]:

$$\Delta\sigma/2 = \left(R_{\infty} + k\right) + X_{\infty}^{(1)} \tanh\left(\gamma^{1} \ \Delta\varepsilon^{\mathrm{p}}/2\right) + X_{\infty}^{(2)} \tanh\left(\gamma^{2} \ \Delta\varepsilon^{\mathrm{p}}/2\right) + X_{\infty}^{(3)} \tanh\left(\gamma^{3} \ \Delta\varepsilon^{\mathrm{p}}/2\right). \tag{8}$$

2.3. Inverse problem and genetic algorithm in parameter identification

The procedure of genetic algorithm consists of three main parts. The first part is system characterization, which means determination of parameters that can completely characterize the system. In the second part, forward modeling, mechanical principles and physical laws are defined to enable prediction of system behaviour. The third part is backward or inverse modeling. Inverse analysis plays an important role in problems where the cause has to be defined from the results. It consists of defining the search methods of unknown sample characteristics by observing sample's response to a probing signal. Definition of objective function represents the solution of inverse problem. The calculating procedure is formulated as:

$$\sigma = \hat{\sigma}(\varepsilon; a_i), \tag{9}$$

where σ and ε are stresses and strains, while a_i = [$X_\infty^{(1)}, X_\infty^{(2)}, X_\infty^{(3)}, \gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}$] are material parameters which have to be identified. Parameter R_∞ is calculated as the difference between initial yield stress and yield stress in stable cycle and therefore isn't part of genetic algorithm calculation procedure.

3. OBJECTIVE FUNCTION

Since evolutionary algorithm for parameter identification is used, the solution of the problem is searched in the global domain. It is not necessary to localize solution domain in order to achieve more accurate data. The chosen objective functions used for comparison in this research are taken in the form published by some authors and also in modified form of each of them as shown in Table 1. The values denoted by asterisk are experimental ones.

Table 1. Objective functions

	Original function		Modified function	
From [6]	$f = \sum_{i=1}^{n} w_i \left[\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)\right]^2 ; w_i = 1$	(10)	$f = \sum_{i=1}^{n} w_i \left[\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)\right]^2 ;\ w_i = 100$	(11)
From [7]	$f = \sum_{i=1}^{n} \left[\frac{\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)}{\sigma_i^*}\right]^2$	(12)	$f = \sum_{i=1}^{n} \left\vert\frac{\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)}{\sigma_i^*}\right\vert$	(13)
From [8]	$f = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left[\frac{\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)}{\sigma_i^*}\right]^2}$	(14)	$f = \sqrt[4]{\frac{1}{n} \sum_{i=1}^{n} \left[\frac{\sigma_i^* - \hat{\sigma}(\varepsilon_i^*; a)}{\sigma_i^*}\right]^2}$	(15)

4. RESULTS AND DISCUSSION

Definitions of objective functions and system characterization are the base for genetic algorithm procedure. Genetic operators for this procedure are adjusted to the particular problem of modeling material behaviour [5], which means identifying material parameters, according to proposed material model. Numerical procedure for materials' parameter identification is performed by application of own developed software solution [9]. Strain–controlled fatigue testing was conducted, following standard procedure [10] and it serves as a base for modeling of material behaviour. Detailed response of the material during cycle loading was recorded during own experiment on circular shaped un-notched specimen. Material parameters in Table 2 have been identified to model material behaviour of the steel 42CrMo4 in normalized state with hardness of 296 HV. The uniaxial test was performed by applying strain with amplitude ε_a = ± 1,5% and mean strain ε_{mean} = 0.

Table 2. Material parameters for presented objective functions, $X_\infty^{(n)}$ (N/mm²), $\gamma^{(n)}$ (-)

Eq.	$X_\infty^{(1)}$	$X_\infty^{(2)}$	$X_\infty^{(3)}$	$\gamma^{(1)}$	$\gamma^{(2)}$	$\gamma^{(3)}$	Eq.	$X_\infty^{(1)}$	$X_\infty^{(2)}$	$X_\infty^{(3)}$	$\gamma^{(1)}$	$\gamma^{(2)}$	$\gamma^{(3)}$
(10)	155	103	83	75	123	683	(11)	170	104	85	70	109	680
(12)	459	78	66	30	182	1089	(13)	157	93	72	114	95	825
(14)	140	54	112	139	1286	107	(15)	89	168	46	91	167	1554

Although seemingly very different parameter values, each group of parameters gives very good solution, as shown in Figure 1 (all curves overlap with experimental one). The difference among experimental response of the material and simulating behaviour $\Delta\sigma$ can not be practically seen. Therefore deviations of simulating stress from experimental ones are calculated. Again, these values are all acceptable. The biggest difference is calculated by using eq. (13) and the calculated value differ from experimental on for only 1,57%.

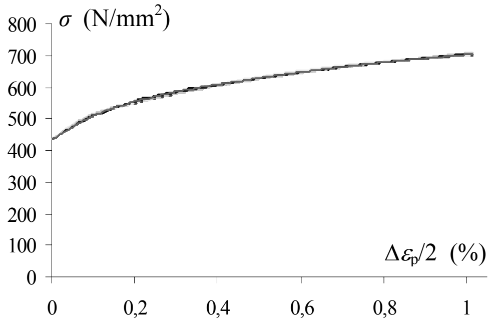

Figure 1.Stress-strain curves in stabile cycle

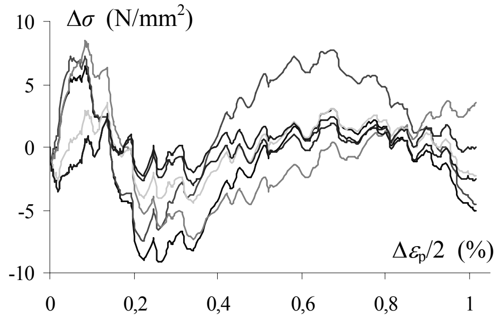

Figure 2.Deviaton of simulating from experimental stress

5. CONCLUSION

Generally, when referring to the functional inverse problems for the parameter identification, appropriate objective function must be used in the most calculation procedures. The choice of the function depends on the numerical procedure in material behaviour modeling that will be used. In genetic algorithm for parameter identification random applications were used to solve complex problem. In order to evaluate robustness in such calculation procedure, considering the choice of objective function, the most commonly used functions were examined, but also their modified versions. The investigation showed extremely good compatibility in results and only very small deviations of simulated from real material's response. Therefore we can conclude that genetic algorithm in parameter identification is robust enough to give reliable data without need to consider the choice of the objective function for inverse problem. The probability to convergence to the accurate results is very high and there is no need for the improvement in the calculation procedure by using specifically oriented objective function.

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