14th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2010, Mediterranean Cruise, 11-18 September 2010

CRITERION FOR SELECTION OF THE WAY FOR CALCULATION OF GEARS

Dr.sc. Sadullah Avdiu, Dr.sc. Nijazi Ibrahimi
Faculty of Mechanical Engineering
Kodra e Diellit pn., 10000 Prishtina
Kosova

ABSTRACT
In this paper the criterion for selection of the way for calculation of gears durability has been analyzed. This criterion reduces the valuable time of designer that he uses during the gears calculation and gives him one of two choices of calculation and verification of safety steps: according to durability against the gears breaking in root or according to durability of gears side pitting. This problem in analytic way has been considered and the graphical presentation of several cases has been reported.
Keywords: Gears durability, gears breaking, Pitting.

1. INTRODUCTION

In the theory of gears, until now, have been developed theoretically and experimentally verified these criteria for gear calculation/design:

- *calculation of the gears transmission under the criteria of durability of the sides of the teeth,*
- *calculation of the gears transmission under the criteria of durability of teeth root,*
- *calculation of transmission of the gears on erosion, and*
- *calculation of transmission of the gears on consumption*.

In practice and for engineering needs mostly are used the first two criteria, while the last two are more for scientific researches, for which the special and expensive equipment is required and also for their application there are certain limitations, which would not be the subject of analysis in this paper.

To day, in the gears calculation/design, as criteria are applied: the sustainability of the tooth root against fracture/breakage and durability of teeth sides under surface destruction/pitting, proving respective safety rate, which must be greater than the allowed minimum.

It is also known that the durability of root and sides of teeth of the gears depends on the dynamic stability of the material, geometry of the tooth profile, the inclination angle of the tooth, the ratio of kinematic transmission of a gearing couple and a range of correction factors *K*, *Y* and *Z*.

However, so far has not been known any method for defining the criteria under which the calculation will be done, but basically, it is calculated the durability of root and side, respectively the safety rates of the gears under the fracture of tooth at root and against surface destruction are verified, a procedure that presents a lot of work, especially knowing that a number of different influential factors must be determined.

According to [3], it is proposed criteria under which a relatively small number of influential factors determine whether to be calculated according to the stability of the tooth's root on breakage or according to durability of the tooth side on Hertz's contact pressure, respectively based on which one of the safety rates will be verified.

2. THEORETICAL BASIS FOR THE CRITERIA DETERMINATION

Criteria for selecting the method of calculating gears presents the report between the needed volume on break/fracture of tooth's root V_F and needed volume on Hertz's contact pressure on teeth sides V_H, expressed as factor:

$$K_V = \frac{V_F}{V_H} \tag{2.1}$$

To define this factor it is started form known expressions of safety rates:

by the tooth root breakage

$$S_F = \frac{2\pi}{10^6}\frac{m_n^2 b n_1 z_1}{P}\frac{Y_x Y_N Y_\delta Y_R Y_{Eht}}{K_A K_v K_{F\alpha} K_{F\beta} Y_{Fa} Y_{sa} Y_\varepsilon Y_\beta \cos\beta}\sigma_{F\,lim} \tag{2.2}$$

and

by destruction of the sides of teeth surface

$$S_H = \frac{m_n z_1}{10^3}\sqrt{\frac{\pi b n_1}{P}}\sqrt{\frac{u}{u+1}}\frac{Z_X Z_L Z_R Z_v Z_W Z_N Z_{Eht}}{\sqrt{K_A K_v K_{H\alpha} K_{H\beta}}Z_H Z_E Z_\varepsilon \cos\beta}\sigma_{H\,lim} \tag{2.3}$$

Multiplying expression (2.2) with $\frac{z_1 \cos\beta}{z_1 \cos\beta}$, while expression (2.3) in quadrate, we get an expression:

$$S_F = \frac{2\pi}{10^6}\frac{m_n^2 z_1^2}{\cos^2\beta}\frac{n_1}{P}b\frac{\cos\beta}{z_1}\frac{Y_x Y_N Y_\delta Y_R Y_{Eht}}{K_A K_v K_{F\alpha} K_{F\beta} Y_{Fa} Y_{sa} Y_\varepsilon Y_\beta}\sigma_{F\,lim} \tag{2.2′}$$

and

$$S_H^2 = \frac{m_n^2 z_1^2}{10^6 \cos^2\beta}\frac{\pi b n_1}{P}\frac{u}{u+1}\frac{(Z_X Z_L Z_R Z_v Z_W Z_N Z_{Eht})^2}{K_A K_v K_{H\alpha} K_{H\beta} Z_H^2 Z_E^2 Z_\varepsilon^2}\sigma_{H\,lim}^2 \tag{2.3′}$$

From (2.2′) we get:

$$\frac{P}{n_1} = \frac{2\pi}{10^6}\frac{m_n^2 z_1^2}{\cos^2\beta}b\frac{\cos\beta}{z_1}\frac{Y_x Y_N Y_\delta Y_R Y_{Eht}}{K_A K_v K_{F\alpha} K_{F\beta} Y_{Fa} Y_{sa} Y_\varepsilon Y_\beta}\frac{\sigma_{F\,lim}}{S_F} \tag{2.4}$$

While from (2.3′):

$$\frac{P}{n_1} = \frac{\pi}{10^6}\frac{m_n^2 z_1^2}{\cos^2\beta}b\frac{u}{u+1}\frac{(Z_X Z_L Z_R Z_v Z_W Z_N Z_{Eht})^2}{K_A K_v K_{H\alpha} K_{H\beta} Z_H^2 Z_E^2 Z_\varepsilon^2}\frac{\sigma_{H\,lim}^2}{S_H^2} \tag{2.5}$$

Left sides of expressions (2.4) and (2.5) are equal, meaning that their right sides are equal as well, from where we get the expression:

$$\frac{2\pi}{10^6}\frac{m_n^2 z_1^2}{\cos^2\beta}b\frac{\cos\beta}{z_1}\frac{Y_x Y_N Y_\delta Y_R Y_{Eht}}{K_A K_v K_{F\alpha} K_{F\beta} Y_{Fa} Y_{sa} Y_\varepsilon Y_\beta}\frac{\sigma_{F\,\mathrm{lim}}}{S_F} =$$

$$= \frac{\pi}{10^6}\frac{m_n^2 z_1^2}{\cos^2\beta}b\frac{u}{u+1}\frac{(Z_X Z_L Z_R Z_v Z_W Z_N Z_{Eht})^2}{K_A K_v K_{H\alpha} K_{H\beta} Z_H^2 Z_E^2 Z_\varepsilon^2}\frac{\sigma_{H\,lim}^2}{S_H^2} \tag{2.6}$$

For cylindrical involute gears with straight teeth where for $\beta = 0°$, $\cos\beta = 1$ and $Y_\beta = 1$, expression (2.6) takes a form:

$$\frac{2}{z_1}\frac{Y_x Y_N Y_\delta Y_R Y_{Eht}}{K_{F\beta} Y_{Fa} Y_{sa} Y_\varepsilon Y_\beta}\frac{\sigma_{F\,lim}}{S_F} = \frac{u}{u+1}\frac{(Z_X Z_L Z_R Z_v Z_W Z_N Z_{Eht})^2}{K_{H\beta} Z_H^2 Z_E^2 Z_\varepsilon^2}\frac{\sigma_{H\,lim}^2}{S_H^2} \tag{2.7}$$

or:

$$V_F \frac{2}{z_1}\frac{Y_x Y_N Y_\delta Y_R Y_{Eht}}{K_{F\beta} Y_{Fa} Y_{sa} Y_\varepsilon Y_\beta}\frac{\sigma_{F\,lim}}{S_F} = V_H \frac{u}{u+1}\frac{(Z_X Z_L Z_R Z_v Z_W Z_N Z_{Eht})^2}{K_{H\beta} Z_H^2 Z_E^2 Z_\varepsilon^2}\frac{\sigma_{H\,lim}^2}{S_H^2} \tag{2.8}$$

With substitution $z_2 / z_1 = u$ in (2.8), the final form of the formula of criteria for teeth calculation of the gears is:

$$\boxed{K_V = \frac{V_F}{V_H} = \frac{1}{2}\frac{S_F}{S_H^2}\frac{\sigma_{H\,lim}^2}{\sigma_{F\,lim}}\frac{z_1 z_2}{z_1 + z_2}\frac{K_{F\beta}}{K_{H\beta}}\frac{Y_{Fa} Y_{sa} Y_\varepsilon}{Z_H^2 Z_E^2 Z_\varepsilon^2}\frac{(Z_X Z_L Z_R Z_v Z_W Z_N Z_{Eht})^2}{Y_x Y_N Y_\delta Y_R Y_{Eht}}} \tag{2.9}$$

3. VALUES OF DIFFERENT FACTORS ACCORDING TO ISO AND DIN STANDARDS

In this paper the cylindrical gears with right teeth of cemented steel are analyzed, where the number of teeth is z_1 = 14 ... 20, the transmission ratio u = 1,0 ... 2,5 and the profile shift coefficient of x_1 is from 0 to 0,5.

Values of factors $K_{F\beta}$, $K_{H\beta}$, Y_δ and Y_R are not taken as in common calculation for this case, but appropriated under preferences by DIN and ISO standards, and values for safety rates are taken those of allowed minimum.

The adopted values are: $S_F / S_H^2 \cong 1{,}2$; $K_{F\beta} / K_{H\beta} \cong 0{,}97$; $Y_\delta = 1{,}05$; Y_R = 0,8685; $Z_R Z_v Z_L = 0{,}92$; Z_{Eht} = 0,95, while values for the factors from (2.9) have value equal to 1.

Therefore, for gears produced from material cemented steel, for which $\sigma_{H\,lim} = 1300$ [N/mm^2] and $\sigma_{F\,lim} = 310$ [N/mm^2], Z_E =2,06·10^5 [N/mm^2], expression (2.9) takes a form:

$$K_V = 0{,}07376763 \frac{z_1 z_2}{z_1 + z_2} \frac{Y_{Fa} Y_{sa} Y_\varepsilon}{Z_H^2 Z_\varepsilon^2} \tag{3.1}$$

Meaning of K_V factor can be defined by the expression (2.1). If with calculation is obtained value for $K_V < 1$ for defined gear, the merit calculation is based on the durability of teeth sides, while for $K_V > 1$ then the calculation should be done according to the sustainability of a breakage of the tooth root.

4. ANALYSIS OF THE OBTAINED RESULTS

In Fig. 1., 2. and 3. are presented the curves of criteria factor K_V for selecting the method of calculating the gears, for three kinematic transmission ratios u = 1.0, 1.7 and 2.5, depending on the number of teeth z_1 and values of the coefficient of profile displacement x_1, for two combinations of sizes of tool in the form of toothed board $h_{a0}^* = 1{,}25$; $\rho_{a0}^* = 0{,}25$ and $h_{a0}^* = 1{,}20$; $\rho_{a0}^* = 0{,}20$, while in fig. 3.4. are presented curves of this factor, depending on the number of teeth z_1 and kinematic transmission ratio u.

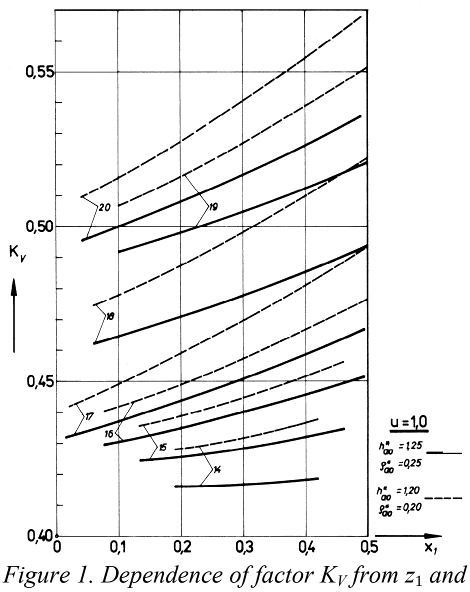

Figure 1. Dependence of factor K_V from z_1 and x_1 for u = 1,0.

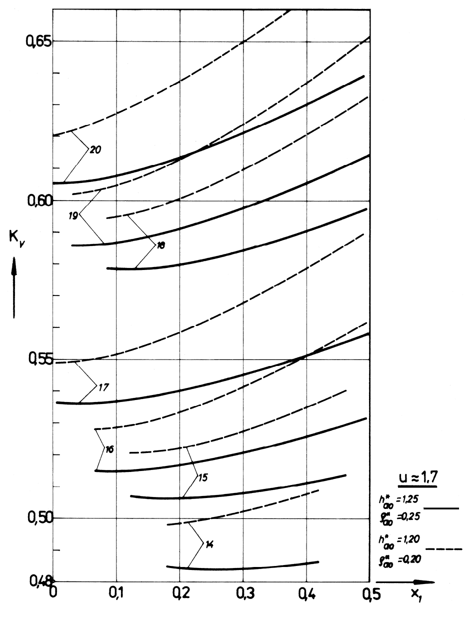

Figure 2. Dependence of factor K_V from z_1 and x_1 for $u \approx 1{,}7$.

From the curves obtained (Fig. 1., 2. and 3.) can be concluded that increasing the number of teeth z_1 and values of the coefficient of profile displacement factor x_1 the values for K_V increase. Also, with increasing the values of transmission kinematic ratio u, the values of this factor increases.

Another characteristics that should be noted at these curves, is that with the increase of transmission kinematic ratio change curve shape. In Fig. 1. curves shows the growth of K_V factor, in Fig. 2. at start growth is lower, while in Fig. 3. at start K_V factor values are greater, then follows reduce to certain values of displacement coefficient.

Changing the sizes of tool in the form of toothed board $h^*_{a0} = 1{,}25$; $\rho^*_{a0} = 0{,}25$ and $h^*_{a0} = 1{,}20$; $\rho^*_{a0} = 0{,}20$ the K_V factor values increase, i.e. increase the sustainability of sides of the teeth, but endanger the sustainability of the tooth root.

According to the curves obtained in Fig. 4. can be concluded that increasing the number of teeth z_1 and transmission kinematic ratio u, the values of the factor of criteria for selecting the method of calculating the gears K_V increase.

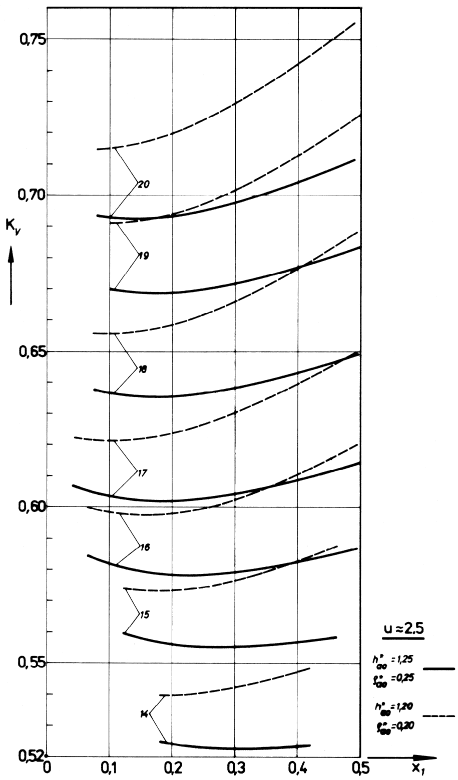

Figure 3. Dependence of factor K_V from z_1 and x_1 for u = 2,5.

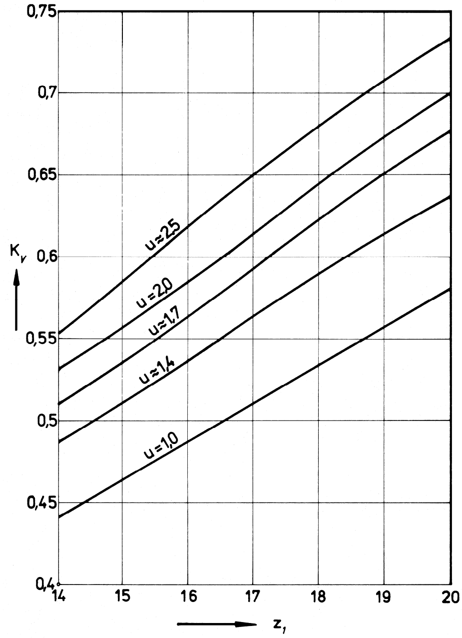

Figure 4. Dependence of factor K_V from z_1 and transmission kinematic ratio u.

5. CONCLUSIONS

Based on the analysis it can be concluded that increasing the number of teeth z_1 and values of the profile shift coefficient x_1 and increased values of transmission kinematic ratio u, K_V factor values increase. Also changing the sizes of the tool in the form of toothed board from $h^*_{a0} = 1{,}25$; $\rho^*_{a0} = 0{,}25$ and $h^*_{a0} = 1{,}20$; $\rho^*_{a0} = 0{,}20$ the values if this factor increases. For interval involved in research, K_V factor values are smaller than 1.0, so for this interval the sides of the teeth are always in danger. So, the calculation of gears sustainability and prove of safety rate is based on the sustainability of sides of the teeth.

6. REFERENCES

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