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SHAFT TORSIONAL OSCILLATION OF INDUCTION MOTOR INCLUDING THE ROTOR EDDY CURRENT WITH AN INERTIA LOAD

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ABSTRACT
The paper present the dynamic modeling of a electrical drive with induction machine with rotor bar eddy currents effects. The computer simulation for the transient operation is obtained from the non-linear differential system of equations, which describe the complete electromechanical system.
Keywords: Dynamic modeling induction motor**,** torsional oscillation, eddy curr*ents*

1. INTRODUCTION

Electric drive is an electromechanical system, which performs the conversion of electrical energy to mechanical energy or vice versa for running various processes such as: production plants, transportation of people or goods, home appliances, pumps, air compressors, computer disc drives, robots, music or image players, etc. To couple electrical motor with mechanical load, the mechanical drives are used. The basic types of mechanical drives are: geared transmission – provides specific fixed type ratios; belt drives - provide flexibility in the positioning of the motor; chain drives – provide infinitely variable speeds; traction drives – provides adjustable speed with relatively high speed. Thus, electromechanical system consists of two parts: electrical and mechanical (Fig. 1). Rotating parts of the motor and mechanical load are the main components of the mechanical part of drive. Usually they have different parameters of the movement, therefore intermediate mechanical chains such as shafts, reducers, belt-drives and screw-drives as well as clutches are used. Always there is a proper parameter of interest, for example, inertia, speed of rotation, elasticity of mechanical chains, characterizing the movement of these parts or entire system. The induction motor are widely used in the industry, has been favoured because of its self starting current, capability and rugged structure low cost and reliability. This paper presents an accurate dynamic model of a squirrel cage induction motor for the transient studies and elastic mechanical system. The machine parameters are not constant and depend on skin effect, saturation and temperature. In this paper will be presented induction motor (IM) model taking into consideration skin

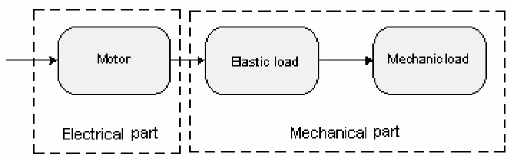

Figure 1. Structure of electromechanical system

effect or sometimes we used as deep bar effect. In an electric drive system the machine is a part of the control system elements. To be able to control the dynamics of the drive system, dynamic behaviour of the machine need to be considered. The dynamic behaviour of IM can be described using dynamic model of IM. The dynamic model considers the instantaneous effects of varying voltages/currents, stator frequency and torque disturbance. In this paper the dynamic model of IM is derived by using d and q variables in a rotating reference frame. Induction motors are commonly controlled by contactors, which are electromagnetic switches that are highly sensitive to voltage depressions and momentary service interruptions. Voltage depressions are huge problems for many industries, and it is probably the most pressing power quality problems today. The Voltage depressions caused by faults on the system affect the performance of induction motors, in terms of the production of both transient currents and transient torques. It is often desirable to minimize the effect of the voltage dip on both the induction motor and more importantly on the process where the motor is used. Large torque peaks may cause damage to the shaft or equipment connected to the shaft. Some common reason for voltage depressions are lightning strikes in power lines, equipment failures, accidental contact power lines, and electrical machine starts.

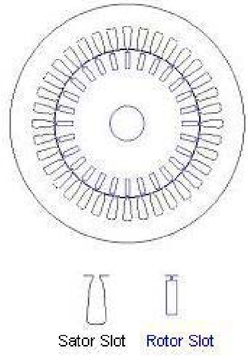

Figure 2. Cross section of IM, stator and rotor

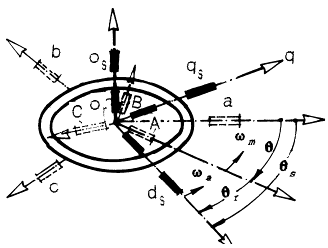

Figure 3. d-q frame in IM

2. MODELLING OF INDUCTION MOTOR

In this paper the mathematical model of the induction motor is modeled based on references [1]. This model is described in space vector formulation in rotating d-q reference frame. This model is obtained considering the induction balanced conditions; the sum of stator currents as well as rotor currents is zero. The d-q model uses two windings for each stator and rotor of the induction motor. A transformation of variables can be used. The power invariant two-axes transformation is used and is defined as: (i) 3-phase to 2-phase (abc-dq) conversion: To convert 3-phase voltages to voltages in the 2- phase synchronously rotating frame, they are first converted to 2-phase stationary frame (α β) or in the movement with an arbitrary and then from the stationary frame to the synchronously rotating frame (dq).

$$u_{d_s} = R_s i_{d_s} + \frac{d\psi_{d_s}}{dt} - \omega_k \psi_{q_s}, \qquad u_{q_s} = R_s i_{q_s} + \frac{d\psi_{q_s}}{dt} + \omega_k \psi_{d_s}$$
$$u_{d_r} = R_r i_{d_r} + \frac{d\psi_{d_r}}{dt} - (\omega_k - \omega)\psi_{q_r}, \quad u_{q_r} = R_r i_{q_r} + \frac{d\psi_{q_r}}{dt} + (\omega_k - \omega)\psi_{d_r} \quad (1),$$

$$\psi_{d_s} = L_s i_{d_s} + L_m i_{d_r}; \quad \psi_{q_s} = L_s i_{q_s} + L_m i_{q_r}$$
$$\psi_{d_r} = L_m i_{d_s} + L_r i_{d_r}; \quad \psi_{q_r} = L_m i_{q_s} + L_r i_{q_r} \quad (2)$$

The electromagnetic torques and rotor equation in d-q frame are:

$$M_{em} = \frac{3}{2} p \frac{L_m}{\sigma L_s L_r} (\psi_{ds}\psi_{qr} - \psi_{qs}\psi_{dr}) \quad (3), \qquad (\frac{1}{p}) J \frac{d\omega}{dt} + M_{load} = M_{em} \quad (4)$$

The dynamic equations in arbitrary reference frame, which is rotating at speed ω_k in the direction of rotor rotation: when $\omega_k=0$, the reference frame is fixed in the stator (q_s-d_s ref. frame); when $\omega_k=\omega_s$, the reference frame is fixed on the synchronously rotating reference frame (ω_1) and when $\omega_k=\omega_r$, the reference frame is fixed in the rotor.

3. MODELIN GOF OF MECHANICAL SYSTEM.

Movement of mechanical part of electromechanical system in general form is described by equations of **Lagrange**-it, [1,3]:

$$\frac{d}{dt}\left(\frac{\partial W_k}{\partial \dot{q}_r}\right) - \frac{\partial W_k}{\partial q_r} + \frac{\partial W_p}{\partial q_r} + \frac{\partial \phi}{\partial \dot{q}_r} = Q_r, \quad (r = 1,2,3,...,n). \quad (5)$$

where $L = W_k - W_p$ – Lagrange's function; Q_i – force, depending on elementary works of external forces and their possible displacement ∂q_i.

4. SKIN EFFECT AND PARAMETERS OF THE ELECTROMECHANICAL SYSTEM

As can be noticed from equations (1-2), the expression for the electromechanical torque (3-4) requires machine parameters. In particular they are: stator resistance R_s, the stator inductivity L_s and magnetizing inductivity L_m, the rotor resistance R_r and inductivity L_r. The parameters can be obtained by basic, straightforward calculations. The stator resistance R_s [Ω] estimation is based on a dc test. To capture the effect of temperature on the stator resistance, before each test, the motor should run with a higher load. The stator inductivity and main inductivity L_m (dependent from saturation) are taken constant. Rotor resistance varies as a function of two factors: slip frequency (through skin effect) and rotor temperature. However, in order to obtain more accurate motor parameters, one has to take into account skin effect in the rotor bars and, to a minor extent, temperature corrections. There are several ways to consider this skin effect [1]. Anyhow, owing to the nature of this effect, the size and shape of the conductors is of importance, therefore, at least the height of the rotor bars should be known in order to be able to correct for the rotor impedance. Pertinently, for the study of skin-effect in rotor bars (Fig.4), the rotor circuit needs to be modified in order to adequately take into account the wide range of frequencies occurring in the machine, respectively a rotor model is presented with n- loops, Fig.5. It has been noted by [1] that the model accuracy increases when the number of the rotor loops is increased to four or more. In this paper, the rotor circuit is modeled as proposed by [1].The model of the the rotor bar, a T-configuration network is used, [1].The rotor bar is divided into n-sections fictive conductors, as shown in Fig.4. The conductors of the bar are connected in parallel with voltage u. The voltage equation for k-bar conductor will be:

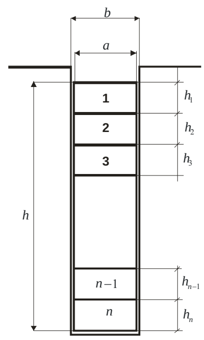

Figure 4. Rotor bar: modeled with n conductor

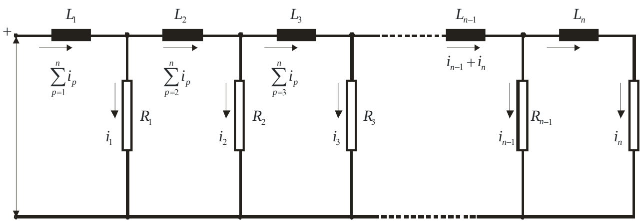

Figure 5.

$$u = R_k i_k + \frac{d\left(\sum_{j=1}^{k} L_j \sum_{p=j}^{n} i_p\right)}{dt} \quad (6) \qquad R_k = \frac{h}{h_k} R_0, \quad k = 1,2,....,n \qquad L_1 = \frac{3}{2}\frac{h_1}{h} L_0, \; L_j = \frac{3}{2}\frac{h_{j-1}+h_j}{h} L_0 \text{ per } j>1 \quad (7).$$

Where is: R_0- resistance of rotor bar without skin effect; L_o –inductivity of bar without skin effect; inductivities L_j, (j=1,2,....,n) is between bar fictive conductors "j-1" (height is small and non-negligible) and "j. The number of the rotor loops "k=1, 2, ..., n" will give the set of differential equations describing the eddy currents in the rotor. The "T" equivalent network for dynamical regime which includes the rotor eddy current is shown in Fig.6. In general case, mechanical systems deal with systems elements of finite stiffness, possible clearance and non-linearity of motor and load characteristics.On base of the equivalent circuits the equations (1-4) describing transient characteristics of IM are in d-q reference frame rotating at synchronous speed $\omega_l=\omega_1$ and in per unit:

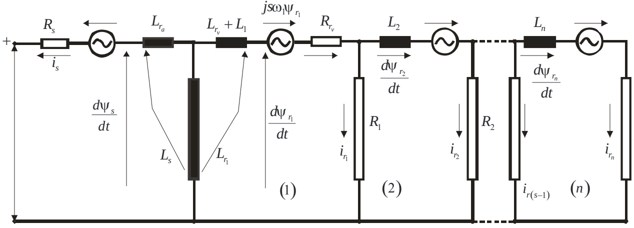

Figure 6. Equivalent sheme of IM including the rotor bar eddy currents.

$$\frac{d\psi_d}{dt} = u_d - \rho_s \psi_d + k_{r1}\rho_s \psi_{1d} + \omega_1 \psi_q, \quad \frac{d\psi_q}{dt} = u_q - \rho_s \psi_q + k_{r1}\rho_s \psi_{1d} - \omega_1 \psi_d$$

$$\frac{d\psi_{1d}}{dt} = k_s \rho_{r1} \psi_d - \rho_{r1} \psi_{1d} + \alpha_2 \psi_{2d} - s\,\omega_1 \psi_{1d}, \quad \frac{d\psi_{1q}}{dt} = k_s \rho_{r1} \psi_q - \rho_{r1} \psi_{1q} + \alpha_2 \psi_{2q} - s\,\omega_1 \psi_{1d}$$

$$\frac{d\psi_{2d}}{dt} = -k_s \rho_{r2} \psi_d + \rho_{r2} \psi_{1d} - \rho_2 \psi_{2d}\, \alpha_3 \psi_{3d} + s\,\omega_1 \psi_{2q}, \quad \frac{d\psi_{2q}}{dt} = -k_s \rho_{r2} \psi_q + \rho_{r2} \psi_{1q} - \rho_2 \psi_{2q} + \alpha_3 \psi_{3q} - s\,\omega_1 \psi_{2d}$$

$$\frac{d\psi_{kd}}{dt} = \beta_{k-1} \psi_{(k-1)d} - \rho_k \psi_{kd} + \alpha_{k+1} \psi_{(k-1)d} + s\,\omega_1 \psi_{kq}, \quad \frac{d\psi_{kq}}{dt} = \beta_{k-1} \psi_{(k-1)q} - \rho_k \psi_{kq} + \alpha_{k+1} \psi_{(k+1)q} - s\,\omega_1 \psi_{kd}$$

$$(k = 3,4,\dots,n)$$

$$\rho_s = \frac{R_s}{L'_s}, \ \rho_{r1} = \frac{R_{r_v} + R_1}{L'_{\gamma_1}}, \ \rho_{r_2} = \frac{R_1}{L'_{\gamma_1}} \quad \rho_k = \frac{R_{k-1} + R_k}{L_k}, \ \alpha_k = \frac{R_{k-1}}{L_k}, \ \beta_k = \frac{R_k}{L_k}, \quad (k = 2,3,\dots,n,)$$

$$L'_s = \sigma L_s, \ L'_{r_1} = \sigma L_{r_1}, \ \sigma = 1 - \frac{L_m}{L_s L_{r_1}}, \quad k_s = \frac{L_m}{L_s}, \ k_{r_1} = \frac{L_m}{L_{r_1}}, \ T_m = J\,\omega_1 \frac{\left(\frac{\omega_b}{p}\right)^2}{P_b} \tag{8}$$

Combining the set of equations for mechanical part (5) and equations (8) which are linked by equation of movement of rotor will give a most general dynamic model for studding of transient electromechanical characteristics of electrical drive with induction motor. To obtain analytical solutions of high order nonlinear differential equations with varying coefficients or sets of those, they must be solved, but it is impossible to solve these equations by any known methods. To get the results, methods of mathematical and computer modeling must be used. By using the optimization technique, reported in [1], the values of the rotor circuit parameters can be found.

5. SIMULATION RESULTS

The digital simulation obtained in the system described with equations (5) and (8), shown in Fig.7, induction starting process and takes in account skin effect.

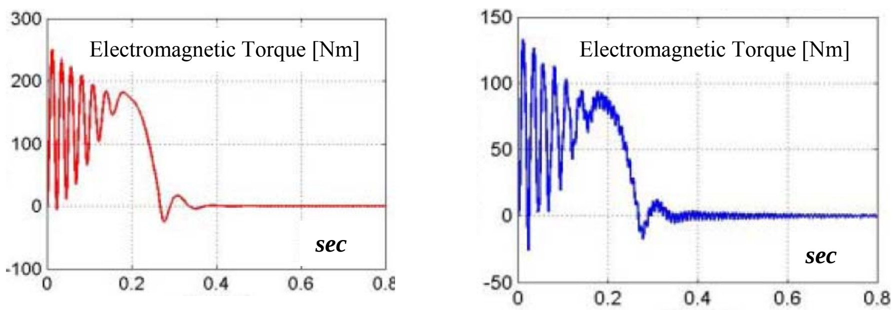

Figure 7. Squirrel-cage induction motor 7.5KW, 340V delta-connected, 50 Hz, 1400 rpm, 9.2 A, four-pole: electromagnetic torque [Nm], without skin effect (left) and with skin effect (right)

6. CONCLUSION

A numerical model of electrical drives with induction motor has been developed to simulate the electromechanical transient characteristics. The developed model, which is represented as a system of differential equations, takes in account the skin effect and elasticity of mechanical system.

7. REFERENCES

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