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MODELING SHAFT DAMPING OF ELECTRICAL DRIVES WITH THREE-PHASE INDUCTION MOTORS WITH SATURATED MAGNETIC CIRCUIT

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ABSTRACT
This paper presents a modelling of transient electromechanical processes of electrical drives with three phase induction motor. During the dynamic processes, the induction machine, as component part of electrical drive, is now exposed to large variations of magnetic stress. To perform a dynamic analysis of induction motor the influence of parameters have to be taken into account by means of a dynamic model that has to combine accuracy with structural simplicity. According to the electromagnetic field theory, a set of differential equations is derived for coupling of the transient process for an electrical machine . A non-dimensional constant coefficient state equation is obtained by transformation. A model has been formulated to include saturation effect of main magnetic circuit of machine. The mechanical system is treated as elastic. The computer program package is developed for digital simulation.
Keywords: *Dynamics of electrical drives, mechanical elastic system*

1. INTRODUCTION
The study and control of mechanical vibrations in electric drives are of high interest in modern industry. In fact, a major preoccupation of today manufacturers is to minimize the mechanical constraints on the drive systems and increase, hence, their reliability and life cycle. The d-q axis theory for induction motor [1] has been generally used to study the motor transients. A direct solution for the electrical drive dynamic equations of operation is very difficult and a digital simulation model is therefore worked out in this paper taking into account the practical approximations which can validate the accuracy of the simulation results.

2. MODELING OF ELECTRCIAL DRIVE WITH INDUCTION MOTOR AND ELASTIC MECHANICAL SYSTEM
A typical drive system is shown in Fig. 1. It consists of an association of four basic functional blocks: the induction motor considered as the central and major part of the system, the mains three-phase electrical source, the mechanical load connected and the end part of the system, and the mechanical transmission link between the motor's rotor and the mechanical load. In this paper the mathematical modelling of an electrical drive consisting from three

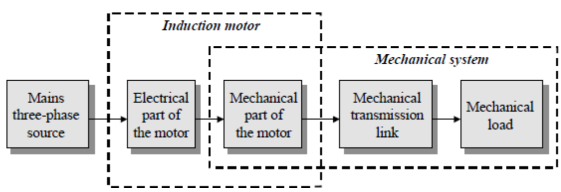


Figure1. Constituent parts of an electric drive system

phase induction motor with short circuit rotor and mechanical system is elastic. The model include: mathematical modelling of electrical part and mathematical modelling of mechanical system. These two models and connected to each other with angular velocity of rotor of electrical machines.

2.1 Mathematical model of induction motor including the saturation

The approach eliminates the redundancy of poly-phase windings, substituting these by their two-axes equivalent, Fig.2. The two axes representation eliminates the mutual magnetic coupling of the phase windings, rendering the magnetic coupling of the phase-winding independent of the current in the other winding. In a second step, both poly-phase windings in the stator and the rotor of an ac machines are viewed from a common frame of reference which is either fixed to the stator, or to the rotor. More generally, the reference frame can be considered rotating at any arbitrary angular velocity ω_k. The common coordinate system is further interpreted as the complex plane, its real axis being denoted as the direct axis (d), and the imaginary axis as the quadrature axis (q-axis). According to Kron [4], a induction motor is symbolically represented by the equivalent circuit Fig.4. The general k-coordinate system rotates at the angular velocity ω_k with respect to the stator windings. The stator voltage equations referred to the k-coordinate system, are expressed in terms of normalized quantities:

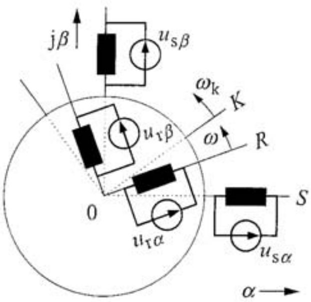


Figure 2. Two-axes representation of an ac

$$u_{sd} = r_s i_{sd} + \frac{d\Psi_{sd}}{d\tau} - \omega_k \Psi_{sq}, \quad u_{sq} = r_s i_{sq} + \frac{d\Psi_{sq}}{d\tau} + \omega_k \Psi_{sd}$$
$$0 = r_r i_{rd} + \frac{d\Psi_{rd}}{d\tau} - (\omega_k - \omega)\Psi_{rq}, \quad 0 = r_r i_{rq} + \frac{d\Psi_{rq}}{d\tau} + (\omega_k - \omega)\Psi_{sd} \quad (1)$$

The angular mechanical velocity of the rotor is ω. As seen from the rotor, the k-coordinate system rotates at ω_k-ω. The flux linkages are proportional to the currents:

$$\Psi_{sd} = l_s i_{sd} + l_m i_{rd}, \quad \Psi_{sq} = l_s i_{sq} + l_m i_{rq}, \quad \Psi_{rd} = l_m i_{sd} + l_r i_{rd}, \quad \Psi_{rq} = l_m i_{sq} + l_r i_{rq} \quad (2)$$

Note that all variables, like currents, voltages and flux linkages, have property of scalars. Time is normalized throughout this paper: $\tau=\omega_{sr}t$, where ω_{sr} is the rated stator frequency. The transformed variables are used to establish the voltage equations of an ac machine [1]:

$$\frac{d\psi_{sd}}{d\tau} = u_{ds} - \frac{r_s}{\sigma l_s}\psi_{ds} + \omega_k \psi_{qs} + \frac{r_s \cdot l_m}{\sigma l_s l_r}\psi_{dr}, \quad \frac{d\psi_{qs}}{dt} = u_{qs} - \frac{r_s}{\sigma l_s}\psi_{q_s} - \omega_k \psi_{ds} + \frac{r_s l_m}{\sigma l_s \cdot l_r}\psi_{qr}$$
$$\frac{d\psi_{dr}}{dt} = -\frac{r_r}{\sigma l_r}\psi_{d_r} + (\omega_k - \omega)\psi_{qr} + \frac{r_r l_m}{\sigma l_s l_r}\psi_{ds}, \quad \frac{d\psi_{qr}}{dt} = -\frac{r_r}{\sigma l_r}\psi_{qr} - (\omega_k - \omega)\psi_{ds} + \frac{r_r l_m}{\sigma l_s l_r}\psi_{qs} \quad (3)$$

in equation (6) $\sigma=1-l_m^2/l_s l_r$; p - number of pair poles, and coordinative system rotate with synchronous speed $\omega_k=\omega_1$. According to Kovács and Rács [3], the space vector theory is used. The equations (1-3) will need to modify to include saturation effect. It is assumed that the rotor is short-circuited and all quantities are related to the stator. Saturation occurs in the expression of the main flux linkage:

$$\Psi_{sd} = l_m i_{dm} + l_{\gamma s} i_{sd}, \quad \Psi_{sq} = l_m i_{qm} + l_{\gamma s} i_{sq}, \quad \Psi_{rd} = l_m i_{dm} + l_{\gamma r} i_{rd}, \quad \Psi_{rq} = l_m i_{qm} + l_{\gamma r} i_{rq} \quad (4)$$

Lm is the magnetization inductance and i_m is the the magnetizing current with components in direction of d, q axes; ψ_m and i_m have the same direction because the iron losses are neglected and consequently. After replacing (4) in (3) we have:

$$u_{d_s} = R_s i_{d_s} + \frac{d(L_m i_{d_s})}{dt} + L_{\gamma_s}\frac{di_{d_s}}{dt} - \omega_k (L_m i_{q_s} + L_{\gamma_s} i_{q_s})\alpha \quad u_{q_s} = R_s i_{q_s} + \frac{d(L_m i_{q_m})}{dt} + L_{\gamma_s}\frac{d\, i_{q_s}}{dt} + \omega_k (L_m i_{d_m} + L_{\gamma_s} i_{d_s})$$
$$u_{d_r} = R_r i_{d_r} + \frac{d(L_m i_{d_m})}{dt} + L_{\gamma_r}\frac{d\, i_{d_r}}{dt} - (\omega_k - \omega)(L_m i_{q_m} + L_{\gamma_r} i_{q_r}) \quad u_{q_r} = R_r i_{d_r} + \frac{d(L_m i_{q_m})}{dt} + L_{\gamma_r}\frac{d\, i_{q_r}}{dt} + (\omega_k - \omega)(L_m i_{d_m} + L_{\gamma_r} i_{d_r}) \quad (5)$$

In the last equations the terms: $d(L_m i_{dm})/(dt)$, $d(L_m i_{qm})/(dt)$, $\omega_k L_m i_{dm}$, $\omega_k L_m i_{qm}$, $(\omega_k-\omega)L_m i_{dm}$ and $(\omega_k-\omega)L_m i_{qm}$, has inductance L_m which depend from saturation of main magnetic circuit. Equations (6) contains effect of saturation, the speed of rotation of coordinate system is ω_k. We assume that leakage flux saturation and main flux saturation can be treated independently. Since only saturation of the main flux path is discussed, leakage inductances are constants. In contrast, the main flux saturation is to be taken into account by means of the machine magnetizing curve, described by the following non-linearity [2]: $\Psi_m = \Psi_m(i_m) = L_m(i_m)\cdot i_m$.

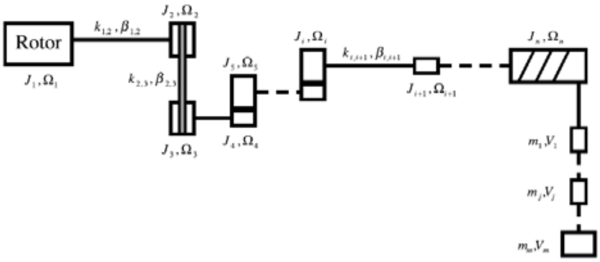


Fig.3

Where in the magnetizing flux linkage as well as the magnetizing inductance change to functions of the magnetizing current variable. The rotation of system ω_k will be equal with the speed of rotation of magnetizing flux $\omega_{m.}$, in this way magnetizing current is unmoveable in relation with coordinate system $i_m = i_{dm} + j\, i_{qm}$. The d axes is coaxial with space vector of magnetizing current and in this way is real quantity: $i_m = i_{dm} = i_m$, dhe $i_{qm} = 0$. Finally having in mind above will be:

$$i_m = i_{d_s} + i_{d_r}, \quad 0 = i_{q_s} + i_{q_r}, \quad \psi_{d_s} = L_m i_m + L_{\gamma_s} i_{d_s}, \quad \psi_{q_s} = L_{\gamma_s} i_{q_s}, \quad \psi_{d_r} = L_m i_m + L_{\gamma_r} i_{d_r}, \quad \psi_{q_r} = L_{\gamma_r} i_{q_r} = -L_{\gamma_r} i_{q_s}$$

$$u_{d_s} = R_s i_{d_s} + \frac{d(L_m i_m)}{dt} + L_{\gamma_s}\frac{d\, i_{d_s}}{dt} - \omega_m i_{q_s} L_{\gamma_s}, \quad u_{q_s} = R_s i_{q_s} + L_{\gamma_s}\frac{d\, i_{q_s}}{dt} + \omega_m(L_m i_m + L_{\gamma_s} i_{d_s}) \tag{6}$$

$$0 = R_r i_{d_r} + \frac{d(L_m i_m)}{dt} + L_{\gamma_r}\frac{d\, i_{d_r}}{dt} - (\omega_m - \omega) L_{\gamma_r} i_{q_s}, \quad 0 = -R_r i_{q_s} - L_{\gamma_r}\frac{d\, i_{q_s}}{dt} + (\omega_m - \omega)(L_m i_m + L_{\gamma_r} i_{d_r})$$

In (6) is respect the condition: $i_{qr}=-i_{qs}$. The speed of rotation of coordinative system ω_m, is obtained by adding the q components of voltages:

$$\omega_m = \frac{u_{q_s} - (R_s - R_r) i_{q_s} - (L_{\gamma_s} - L_{\gamma_r})\frac{d\, i_{q_s}}{dt}}{2 L_m i_m + i_{d_s}(L_{\gamma_s} - L_{\gamma_r}) + L_{\gamma_r} i_m} + \frac{\omega(L_m i_m + L_{\gamma_r} i_m - L_{\gamma_r} i_{d_s})}{2 L_m i_m + i_{d_s}(L_{\gamma_s} - L_{\gamma_r}) + L_{\gamma_r} i_m} \tag{7}$$

Equation in d direction contain term $d(L_m i_m)/(dt)$. Since quantities $\psi_m = L_m i_m$, i_m and L_m are changeable is helpful the derivate to present in appropriate form: $\frac{d(L_m i_m)}{dt} = \frac{d\,\psi_m}{dt} = \frac{d\,\psi_m}{d\, i_m} = \frac{d\, i_m}{dt}$. In voltage equations in q directions appears the saturated value of magnetization inductance L_m, and in q direction transient saturated value of magnetization inductance, dynamic inductivity L, defined as $L=d\psi_m/di_m=L(i_m)$.

From known magnetization curve $\psi_m=F(i_m)$, we can determine dynamic inductivity L for each value of current i_m, as ratio $L=d\psi_m/di_m$. After been determined dynamic inductivity L, equations (6) will be:

$$\frac{d\, i_m}{dt} = \frac{u_{d_s} - (R_s - l R_r) i_{d_s} - l R_r i_m + \omega L_{\gamma_s} i_{q_s}}{l(L + L_{\gamma_r}) + L}, \qquad \frac{d\, i_{q_s}}{dt} = \frac{u_{q_s} - R_s i_{q_s} - \omega_m(\psi_m + i_{d_s} L_{\gamma_s})}{L_{\gamma_s}}$$

$$\frac{d\, i_{d_s}}{dt} = \frac{u_{d_s} - (R_s + \frac{L}{L + L_{\gamma_r}} R_r) i_{d_s}}{L'_s} + \frac{\frac{L}{L + L_{\gamma_r}} R_r i_m + \omega_m i_{q_s} L'_s - \omega i_{q_s}(L'_s - L_{\gamma_s})}{L_s'} \tag{8}$$

Where is $l=L_{\gamma s}/L_{\gamma r}$, transient stator inductivity $L'_s=L_{\gamma s}+L_{\gamma r}/(L+L_{\gamma r})$, the speed ω_m as function of machine parameters will be: $\omega_m = \frac{u_{q_s} - (R_s - l R_r) i_{q_s} + l\,\omega[\psi_m + (i_{m_i} - i_{d_s}) L_{\gamma_r}]}{(1 + l)\psi_m + L_{\gamma_s} i_m}$. The electromagnetic torque is $M_{em} = \frac{3}{2} p\, \psi_m i_{q_s}$. In equations (7-8) and equations for speed, appears two nonlinear functions: $\psi_m = F(i_m)$ $L = F(i_m)$ which will be presented analytically [1].System of equations (6-8) together with rotor equations present the dynamic mathematical model of induction motor which include saturation of main magnetic circuit.

3. MODELLING OF THE MECHANICAL SYSTEM

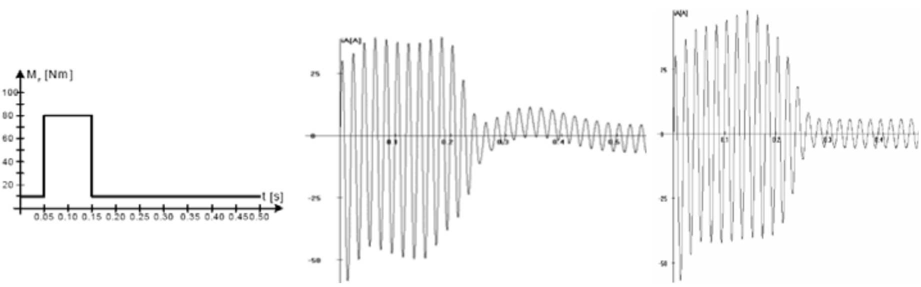
In general the mechanical system, Fig. 3, will be treated with n-stage coordinates and based on equations of **Lagrange**-it, of second or.der [1], will find a set of differential equations which describe the non-stationary motion of mechanical system in the matrix form:

$$[J^*][\ddot{q}]+[b^*][\dot{q}_1]+[c^*][q_2]=[M^*] \quad (9)$$

where is: [J*] - matrix of moment of inertia; [b*]-matrix of dumping coefficients, [c*]- matrix of stiffness coefficients, $[\ddot{q}]$-colum matrix of generalised acceralation and [M*]-matrix of perturbation torque. Matrix equation (9) together with system of differential equation of induction motor (1-8) represent the general mathematic model of electric drives system.. .

4. ANALYSIS OF CASE STUDY

It is analyzed the process of starting with short overload for asynchronous motor of 4 kW, Fig.4a, with the parameters: R_s =1,16 Ω , R_r = 4.74 Ω , $L_{\gamma\sigma}$ = 0,024 H, $L_{\gamma r}$ = 0,0034H, p = 2, J = 0,048 kgm2. The magnetic characteristic is approximated in the form:$\Psi_m(i_m)$=04 i_m for i_m<0.75 A and $0.407 \cdot 0.921^{Im} \cdot i_m^{0.81}$ *for* i_m*>0.75A.* The short mechanical overload is presented in the figure Fig.4a and b. The difference between the plots models is evident. The exact method emphasizes the existence of an oscillation of low frequency superposed on the transient evolution of the current.



a/ b/ c/

Figure 4. a/The short mechanical overload; current in phase A: b/ without saturation, b/saturation

5. CONCLUSIONS

On basis of electromagnetic and mechanical theory is formed an mathematical model of electric drives system with induction machines with mechanical elastic system involving the dynamic characteristics of the induction motor for investigations of transient electromechanical processes. The mathematical model of induction motor has included the saturation of main magnetizing inductivity. The computer program package is developed for digital simulation..

6. REFERENCES

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