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FORMS AND MODELING OF NON-STATIONARY WATER VAPOR DIFFUSION BY A FLAT WALL

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ABSTRACT
In this paper is analyzed and modeled the non-stationary diffusion of water vapor through the flat wall. Balance equation of moisture respectively the water vapor flux between the two sections, for the given layer, is specified by the Laplace's transformation. Fluxes of water vapor (interior and exterior fluxes) are expressed on both sides of the wall surface with thermotechnics relevant features, according to the partial pressure, to concentration of water vapor and in order to absolute humidity. In analytical and graphical way is represented the changing of non-stationary flux of water vapor in the function of external temperature, wall thickness, relative resistance factor of water vapor and of outside relative humidity air.
Keywords: diffusion, flux, water vapor, wall, humidity.

1. NONSTATIONARY DIFFUSION OF WATER VAPOR THROUGH THE FLAT WALL

Partial pressure of water vapor p_w, by the nonstationary diffusion of moisture through the wall changes in function of time τ and distance x from the wall surface, respectively: $p_w = p_w(\tau,x)$. This applies only if the surface of the wall has not gradient of partial pressure and if the wall is built from homogeneous material. Otherwise, partial pressure of water vapor to the wall depends of the coordinates y and z. In fig. 1 is presented onedimensional nonstationary diffusion of water vapor respectively the specific flux of water vapor through the flat wall. Within the wall layer is separated a layer with thickness dx [1].

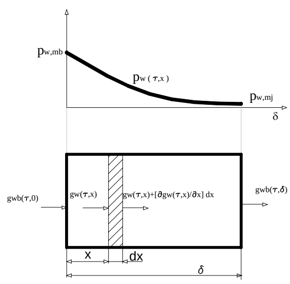

Figure 1. Nonstationary diffusion of water vapor through the flat wall

Equation of moisture equilibrium (the change of water vapor flux between two sections) for the given layer (as in fig. 1), is:

$$g_w(\tau,x)-\left[g_w(\tau,x)+\left(\frac{\partial g_w(\tau,x)}{\partial x}\right)dx\right]=\frac{\partial}{\partial\tau}\left[\frac{\mu}{D}p_w(\tau,x)dx\right] \quad \text{... (1)}$$

or $$g_w(\tau,x)-\left[g_w(\tau,x)+\left(\frac{\partial g_w(\tau,x)}{\partial x}\right)dx\right]=\frac{\partial}{\partial\tau}\left[c_w(\tau,x)dx\right]=\frac{\partial}{\partial\tau}\left[c_L\omega_w(\tau,x)dx\right]$$

Where: p_w, Pa - partial pressure of water vapor through the layer of the wall, c_L, kg/m^3 and c_W, kg/m^3 - concentrations of dry air and moisture; μ, kg/(msPa) – permeability coefficient of water vapor in the wall; and D, m^2/s - diffusion coefficient of water vapor through the wall.

Since, μ, D dhe c_L do not change with the time then from equation (1) appears [2]:

$$-\frac{\partial g_w(\tau,x)}{\partial x}=\frac{\mu}{D}\frac{\partial p_w(\tau,x)}{\partial\tau}=\frac{1}{R_wT}\frac{\partial p_w(\tau,x)}{\partial\tau}=\frac{\partial c_w(\tau,x)}{\partial\tau}=c_L\frac{\partial\omega_w(\tau,x)}{\partial\tau} \quad \text{... (2)}$$

For the examined elementary layer of the wall we have:

$$g_w(\tau,x)=-\mu\frac{\partial p_w(\tau,x)}{\partial x}=-D\frac{\partial c_w(\tau,x)}{\partial x}=-\lambda_d\frac{\partial\omega_w(\tau,x)}{\partial x} \quad \text{... (3)}$$

Partial differential equations (2) and (3) represent mathematical models of nonstationary diffusion water vapor through the wall, respectively, determine the specific flux of water vapor g_w through the wall, in function of time τ and distance x from the surface of the wall.
To reckon the solutions of $g_w(\tau,x)$ and $p_w(\tau,x,)$ respectively $c_w(\tau,x,)$ and $\omega_w(\tau,x,)$, first we use the Laplace's transformations depending of parameter τ. By zero initial conditions, equations (2) and (3) represent the system of two ordinary differential equations by parameter x, and transformed into the form:

$$-\frac{dG_w(s,x)}{dx}=\frac{\mu}{D}sP_w(s,x)=\frac{1}{R_wT}sP_w(s,x)=sC_w(s,x)=c_LsW_w(s,x) \quad \text{... (4)}$$

$$G_w(s,x)=-\mu\frac{dP_w(s,x)}{dx}=-D\frac{dC_x(s,x)}{dx}=-\lambda_d\frac{dW_w(s,x)}{dx} \quad \text{... (5)}$$

Where:

$G_w(s,x)$, $P_w(s,x)$, $C_w(s,x)$ and $W_w(s,x)$ are variables of Laplace's transformations by $g_w(\tau,x)$, $p_w(\tau,x)$, $c_w(\tau,x)$ and $\omega_w(\tau,x)$.

After replacing the integration constants of and required transformations, from the last equations appears:

$$G_w(s,x)=-\mu P_{w,mb}(s)k\sqrt{s}sh(k\sqrt{s}x)+G_{w,mb}(s)ch(k\sqrt{s}x) \quad \text{... (6)}$$

$$P_w(s,x)=P_{w,mb}(s)ch(k\sqrt{s}x)-\frac{G_{w,mb}(s)}{\mu k\sqrt{s}}sh(k\sqrt{s}x) \quad \text{... (7)}$$

Where: $sh(k\sqrt{s}x)$ and $ch(k\sqrt{s}x)$ - hyperbolic functions.
Equation (6) describes the change of diffusion flux of water vapor that passes through the wall, while equation (7) the changing of the partial pressure of water vapor through the wall.

2. FORMS AND MODELING OF NON-STATIONARY WATER VAPOR DIFFUSION BY A FLAT WALL

If we use into the equations (6) and (7) the boundary conditions which influence on the external surface of the wall, ie for x = δ we have $G_W(s, \delta) = G_{w, mj}(s)$ and $P_W(s, \delta) = P_{w, mj}(s)$. Also if, in the system of equations (6) and (7), we add the equations of convection water vapor from the inside and from outside the wall [3]:

$$G_{w,mb}(s)=\beta_b\left[P_{wb}(s)-P_{w,mb}(s)\right] \quad \text{... (8)}$$

$$G_{w,mj}(s) = \beta_j \left[P_{w,mj}(s) - P_{wj}(s) \right] \quad \text{... (9)}$$

So, from the above equations, we achieved the mathematical models of nonstationary diffusion of water vapor through the wall between indoor air and wall, and between the wall and outside air. Respectively, water vapor flux through the wall from the indoor air to outdoor air:

$$G_{w,mb}(s) = \frac{k_d(T_b s + 1)}{T_n s + 1} P_{wb}(s) - \frac{k_d}{T_n s + 1} P_{wj}(s) \quad \text{... (10)}$$

where: $T_n = \frac{1}{D\left[\delta\beta_b\beta_j + \mu\left(\beta_b + \beta_j\right)\right]}\left[\mu^2\delta + \frac{\mu(\beta_b+\beta_j)\delta^2}{2} + \frac{\beta_b\beta_j\delta^3}{6}\right]$ - time constant, s;

$T_b = \frac{1}{D}\left(\frac{\delta^2}{2} + \frac{\mu\delta}{\beta_j}\right)$ - time constant, s; $k_d = \frac{1}{\frac{1}{\beta_b} + \frac{\delta}{\mu} + \frac{1}{\beta_j}}$ - confirmation constant, $kg/(m^2 sPa)$

Equation (10), respectively the water vapor flux, can be used also [4]:

- In function of moisture concentration c_w if we replace $\mu = D$, $p_w = c_w$, $\beta_b = \gamma_b$, $\beta_j = \gamma_j$;
- In function of absolute humidity ω_w if we replace $\mu = \lambda_d$, $p_w = \omega_w$, $\beta_b = \alpha_{db}$, $\beta_j = \alpha_{d\,j}$.

In Table 1 are given forms and models of non-stationary water vapor diffusion by a flat wall, respectively the expressions for water vapor flux of the wall with its features, in view of the partial pressure p_w, in view of the water vapor concentration c_w, and in function of absolute humidity w_ω.

Table 1. Forms and models of non-stationary water vapor diffusion by a flat wall

In function of p_w, Pa	*In function of* c_w, kg/m^3	*In function of* ω_w, kg/kg
$G_{w,mb}(s) = \frac{k_d(T_b s+1)}{T_n s+1} P_{wb}(s) - \frac{k_d}{T_n s+1} P_{wj}(s)$	$G_{w,mb}(s) = \frac{k_d(T_b s+1)}{T_n s+1} C_{wb}(s) - \frac{k_d}{T_n s+1} C_{wj}(s)$	$G_{w,mb}(s) = \frac{k_d(T_b s+1)}{T_n s+1} W_{wb}(s) - \frac{k_d}{T_n s+1} W_{wj}(s)$
$T_n = \frac{1}{D\left[\delta\beta_b\beta_j + \mu\left(\beta_b+\beta_j\right)\right]}\left[\mu^2\delta + \frac{\mu(\beta_b+\beta_j)\delta^2}{2} + \frac{\beta_b\beta_j\delta^3}{6}\right]$	$T_n = \frac{1}{D\left[\delta\gamma_b\gamma_j + D\left(\gamma_b+\gamma_j\right)\right]}\left[D^2\delta + \frac{D(\gamma_b+\gamma_j)\delta^2}{2} + \frac{\gamma_b\gamma_j\delta^3}{6}\right]$	$T_n = \frac{1}{D\left[\delta\alpha_{d,b}\alpha_{d,j} + \lambda_d\left(\alpha_{d,b}+\alpha_{d,j}\right)\right]}\left[\lambda_d^2\delta + \frac{\lambda_d(\alpha_{d,b}+\alpha_{d,j})\delta^2}{2} + \frac{\alpha_{d,b}\alpha_{d,j}\delta^3}{6}\right]$
$T_b = \frac{1}{D}\left(\frac{\delta^2}{2} + \frac{\mu\delta}{\beta_j}\right)$	$T_b = \frac{1}{D}\left(\frac{\delta^2}{2} + \frac{D\delta}{\gamma_j}\right)$	$T_b = \frac{1}{D}\left(\frac{\delta^2}{2} + \frac{\lambda_d\delta}{\alpha_{d,j}}\right)$
$k_d = \frac{1}{\frac{1}{\beta_b} + \frac{\delta}{\mu} + \frac{1}{\beta_j}}$	$k_d = \frac{1}{\frac{1}{\gamma_b} + \frac{\delta}{D} + \frac{1}{\gamma_j}}$	$k_d = \frac{1}{\frac{1}{\alpha_{d,b}} + \frac{\delta}{\lambda_d} + \frac{1}{\alpha_{d,j}}}$

3. ANALYSIS OF NON-STACIONARY WATER VAPOR DIFFUSION BY A FLAT WALL

In view of the upper expressions, by means of the simulations respectively the diagrams which are presented in continuity, it is analyzed the non-stationary water vapor diffusion by a flat wall.

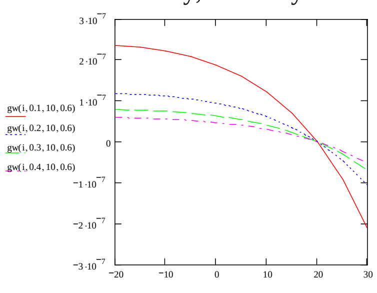

Figure 2. Changing of water vapor flux, kg/(m^{2}s), for cases where δ= 0.1, 0.2, 0.3, 0.4; μ_d = 10 and χ = 0.6, in function of temperature

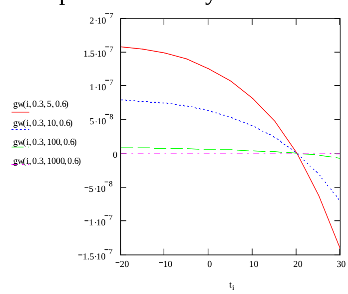

Figure 3. Changing of water vapor flux, kg/(m^{2}s), for cases where δ=0.3, μd=5; 10; 100; 1000 dhe χ=0.6, in function of temperature

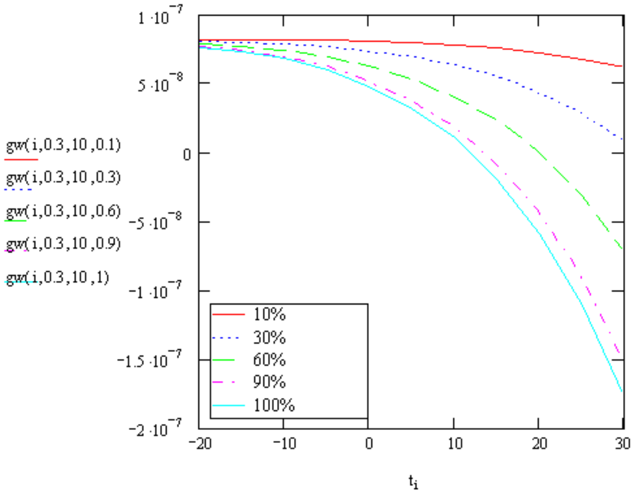

Figure 4. Changing of water vapor flux, kg/(m²s), for cases where δ=0.3, μd=10 dhe χ=0.1; 0.3; 0.6; 0.9; 1, in function of temperature

4. CONCLUSION

In this paper is given an analogy of the water vapor transmission through the flat wall by heat transmission. However, the basis of this paper lies in the formulation of analytical expressions of non-stationary diffusion of water vapor, the adducting of the modeling and presentation of water vapor flux in different forms, including energy and mass convection and conduction through the flat wall. The importance of analytical expressions is given by the above diagrams (figures 2, 3, and 4) where the practice part is reflected to the analysis of thermal flux by changing parameters such as thickness of the wall, relative humidity and permeability coefficient of materials.

5. REFERENCES

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