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THE STRAIN INTENSITIES

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ABSTRACT
In the paper are defined the normal strain intensity, sometimes a reduced strain or strain intensity, and the shearing strain intensity. The shearing strain intensity is proportional to the quadratic invariant of the strain deviator, which may be regarded as an overall characteristic of the distortion of an element. The normal strain intensity corresponds to the elastic energy of distortion apart from a constant factor.
Analyses is worked the different vector space. The equation of the deviatoric plane is given in the isotropic vector space of the normal strain as and the isotropic vector space of shear strain. In the same *manner are given the component parts of the drviatoric strain vector in these isotropic vector spaces*.
Keywords: deviatoric plane, normal strain intensity, shearing strain intensity.

1. INTRODUCTION
In the homogenous vector space a strain vector is decomposed on the normal strain and shear strain component. The normal strain is defined as the normal strain on a plane, and shear strain is defined as the shear strain in a plane. In the homogenous and isotropic vector space, the component of the normal strain is transformed to the mean normal strain and the component of the shear strain is transformed to the deviatoric vector component.
The component parts of the deviatoric strain vector are normal and tangential components. The normal component has the geometrically representation in the vector space of the normal strains and tangential component in the vector space of the shearing strains.

2. THE STRAIN VECTOR IN THE HOMOGENOUS SPACE
The deformation vector Δ **b** = $\overrightarrow{Q_0'Q_1}$ = $\overrightarrow{P_1Q_1}$ - $\overrightarrow{P_1Q_1'}$ is a lineal vector function A of the vector **b** = $\overrightarrow{P_0Q_0}$ [1], i.e.

$$\Delta \mathbf{b} = A\,\mathbf{b} = b\,A\,\mathbf{n}, \qquad (1)$$

into two components transformations: one of these corresponds to a rigid body motion $\overrightarrow{Q_0'Q}$ = $b\,C$ **n**; the other, which we have termed pure deformation $\overrightarrow{QQ_1}$ = $\overrightarrow{Q_0'Q_1'}$ = $b\,B$ **n** = $\overrightarrow{Q_0'R}$ + $\overrightarrow{RQ_1'}$ = $b\,B_I$ **n** + $\overrightarrow{RQ_1'}$.

The strain vector $b\,B$ **n** can be decomposed on the normal strain $\boldsymbol{\varepsilon}_n$ = $\overrightarrow{Q_0'Q_1''}$ and the tangential (shear) strain $\boldsymbol{\varepsilon}_t$ = $\overrightarrow{Q_1''Q_1'}$ (Figure 1). The magnitude ε_n of the normal component $\boldsymbol{\varepsilon}_n$ can be written as (Figure 1)

$$b\,\varepsilon_n = \overrightarrow{Q_0^{'}R}\cdot \mathrm{n} = b\,B_1\,\mathbf{n}\cdot\mathbf{n} = b\,(\sum_{i=1}^{3}\cos\alpha_{in}\varepsilon_{ii}\vec{e}_i\,)\cdot(\sum_{i=1}^{3}\cos\alpha_{in}\vec{e}_i\,) = b\,\sum_{i=1}^{3}(\cos\alpha_{in})^2\,\varepsilon_{ii} \qquad (2)$$

or, the normal strain vector

$$b\,\boldsymbol{\varepsilon}_{\mathrm{n}} = b\,\varepsilon_n\,\mathbf{n} = b\;\varepsilon_n\,(\sum_{i=1}^{3}\cos\alpha_{in}\vec{e}_i\,). \qquad (3)$$

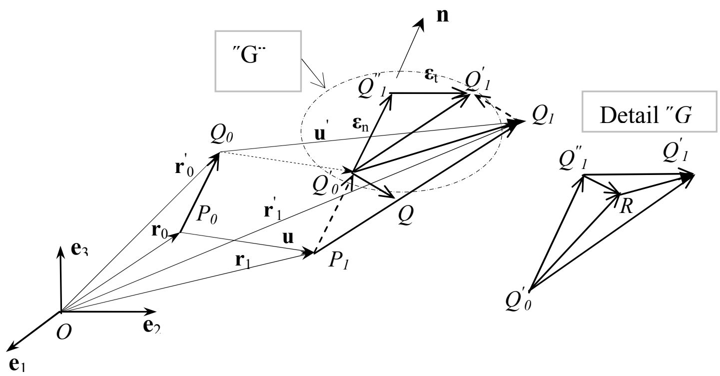

Figure 1. Decomposition the deformation vector

The shear strain $\boldsymbol{\varepsilon}_{\mathrm{t}}$ is defined by (Figure 1)

$$b\,\boldsymbol{\varepsilon}_{\mathrm{t}} = \overrightarrow{Q_1^{''}Q_1^{'}} = \overrightarrow{Q_1^{''}R} + \overrightarrow{RQ_1^{''}} = b\,\sum_{i=1}^{3}\cos\alpha_{in}(\varepsilon_{ii} - \varepsilon_n)\vec{e}_i + b\,D_2\,\mathbf{n}$$

(4)

where $D_1\mathbf{n} = \sum_{i=1}^{3}\cos\alpha_{in}(\varepsilon_{ii} - \varepsilon_n)\vec{e}_i$ [2].

3. THE STRAIN VECTOR IN THE HOMOGENOUS AND ISOTROPIC SPACE

In the homogenous and isotropic vector space of the normal strain is $\cos\alpha_{1n} = \cos\alpha_{2n} = \cos\alpha_{3n} = \frac{1}{\sqrt{3}}$ and is worth

$$\sqrt{3}\,B_1\,\mathbf{n} = \varepsilon_{11}\,\mathbf{e}_1 + \varepsilon_{22}\,\mathbf{e}_2 + \varepsilon_{33}\,\mathbf{e}_3. \qquad (5)$$

Accordingly, in the isotropic vector space of the normal strain, where a point can be identity by components of the normal strain P (ε_{11}, ε_{22}, ε_{33}), the strain vector $B\,\mathbf{n}$ ″is projected″ to the vector $\sqrt{3}\,B\,\mathbf{n}$. The vector (3) has the geometrical presentation in this place [3], i.e.

$$\sqrt{3}\,\boldsymbol{\varepsilon}_{\mathrm{n}} = \sqrt{3}\;\varepsilon_n\,\mathbf{n} = \varepsilon_n\,(\,\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3). \qquad (6)$$

The vector

$$\sqrt{3}\;D_1\mathbf{n} = \sqrt{3}\;(B\,\mathbf{n} - \boldsymbol{\varepsilon}_{\mathrm{n}}) = (\varepsilon_{11} - \varepsilon_n)\,\mathbf{e}_1 + (\varepsilon_{22} - \varepsilon_n)\,\mathbf{e}_2 + (\varepsilon_{33} - \varepsilon_n)\,\mathbf{e}_3 \qquad (7)$$

lies on the plane of the unit vector $\mathbf{n}$, well is $\sqrt{3}\,(B\,\mathbf{n} - \boldsymbol{\varepsilon}_{\mathrm{n}})\cdot\mathbf{n} = 0$, that given

$$(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) - 3\,\varepsilon_n = 0. \qquad (8)$$

By (14) is defined the Hessian normal form the equation of the plane. The plane (8) is cooled the deviatoric plane, and $\sqrt{3}\;\varepsilon_n \geq 0$ is the distance of the plane from is the origin.

The matrix **B** of the linear operator B is symmetric, i.e. $\varepsilon_{ij} = \varepsilon_{ji}$ if $i \neq j$. Accordingly, in the homogenous and isotropic vector space of the shear stresses, the component $D_2\,\mathbf{n}$ is given by [4]

$$D_2\,\mathbf{n} = \frac{1}{\sqrt{3}}\,[\varepsilon_{12}\,(\mathbf{e}_1 + \mathbf{e}_2) + \varepsilon_{23}\,(\mathbf{e}_2 + \mathbf{e}_3) + \varepsilon_{31}\,(\mathbf{e}_3 + \mathbf{e}_1)]$$

or

$$D_2\,\mathbf{n} = \sqrt{\frac{2}{3}}\,(\varepsilon_{23}\,\mathbf{f}_1 + \varepsilon_{31}\,\mathbf{f}_2 + \varepsilon_{12}\,\mathrm{f}_3) \qquad (9)$$

where $\mathbf{f}_1 = \frac{\sqrt{2}}{2}\,(\mathbf{e}_2 + \mathbf{e}_3)$, $\mathbf{f}_2 = \frac{\sqrt{2}}{2}\,(\mathbf{e}_3 + \mathbf{e}_1)$ i $\mathbf{f}_3 = \frac{\sqrt{2}}{2}\,(\mathbf{e}_1 + \mathbf{e}_2)$. In the orthonormal base ($\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{f}_3$) the vector $\mathbf{n}$ is defined by

$$\mathbf{n} = (\mathbf{n}\cdot\mathbf{f}_1)\,\mathbf{f}_1 + (\mathbf{n}\cdot\mathbf{f}_2)\,\mathbf{f}_2 + (\mathbf{n}\cdot\mathbf{f}_3)\,\mathbf{f}_3 \qquad (10)$$

where $\mathbf{n}\cdot\mathbf{f}_1 = \mathbf{n}\cdot\mathbf{f}_2 + \mathbf{n}\cdot\mathbf{f}_3 = \frac{1}{\sqrt{3}}$.

The strain vector $B\,\mathbf{n}$ in the vector space of the shear stress, where a point P can be identify by the values of the shearing stresses, i.e. $P = (\varepsilon_{12}, \varepsilon_{23}, \varepsilon_{31})$, is ″projected″ to the vectors $\sqrt{\frac{3}{2}}\,D_2\,\mathbf{n}$. The vector $\sqrt{\frac{3}{2}}\,D_2\,\mathbf{n}$ lies on the plane of the unit normal $\mathbf{n}$, therefore is $\sqrt{\frac{3}{2}}\,D_2\,\mathbf{n}\cdot\mathbf{n} = 0$, and this given

$$\varepsilon_{12} + \varepsilon_{23} + \varepsilon_{31} = 0. \qquad (11)$$

By (17) is determined the Hessian normal form the equation of the plane in the vector space of the shearing stresses. The plane (11) goes through the origin of the orthonormal base ($\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{f}_3$).

4. THE NORMAL STRESS INTENSITY

The deviatoric component $\sqrt{3}\;D\,\mathbf{n} = \sqrt{3}\;D_1\,\mathbf{n} = \sqrt{3}\;(B\,\mathbf{n} - \boldsymbol{\varepsilon})$ of the vector $B\,\mathbf{n}$, in the vector space of the principal normal strain, is given by

$$\begin{aligned}\sqrt{3}\;D\,\mathbf{n} &= \sqrt{3}\;D_1\,\mathbf{n} = \sqrt{3}\,(B\,\mathbf{n} - \boldsymbol{\varepsilon})\\ &= (\varepsilon_1 - \varepsilon)\,\mathbf{e}'_1 + (\varepsilon_2 - \varepsilon)\,\mathbf{e}'_2 + (\varepsilon_3 - \varepsilon)\,\mathbf{e}'_3.\end{aligned} \qquad (12)$$

The vector (12) lies on the plane of the unit normal vector $\mathbf{n} = (\mathbf{n}\cdot\mathbf{e}'_1)\,\mathbf{e}'_1 + (\mathbf{n}\cdot\mathbf{e}'_2)\,\mathbf{e}'_2 + (\mathbf{n}\cdot\mathbf{e}'_3)\,\mathbf{e}'_3$, therefore is $\sqrt{3}\,(B\,\mathbf{n} - \boldsymbol{\varepsilon})\cdot\mathbf{n} = 0$, and this given

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 - 3\,\varepsilon = 0. \qquad (13)$$

By (19) is determined the Hessian normal form the equation of the plane in the vector space of the principal normal stresses. The plane (19) is cooled the deviatoric plane, and $\sqrt{3}\;\varepsilon \geq 0$ is the distance of the plane from is the origin. If $\varepsilon_n = \varepsilon = 0$ in (13), then the plane (17) goes through the origin of the orthonormal base ($\mathbf{e}'_1$, $\mathbf{e}'_2$, $\mathbf{e}'_3$).

In the process uniaxial tension, if $\varepsilon_2 = \varepsilon_3 = -\frac{1}{2}\varepsilon_1$ and $\varepsilon = 0$, from (12) is obtained

$$\sqrt{3}\,|D\,\mathbf{n}| = \sqrt{\frac{3}{2}}\varepsilon_1. \qquad (14)$$

The quantity

$$\varepsilon_1 = \varepsilon_i = \sqrt{2}\;|D\,\mathbf{n}| \qquad (15)$$

is called the normal stress intensity. The magnitude of the $|D\,\mathbf{n}|$, according to (7) and (9), is

$$\begin{aligned}|D\,\mathbf{n}| &= (|B_1\,\mathbf{n} - \boldsymbol{\varepsilon}|^2 + |D_2\;\mathbf{n}|^2)^{1/2}\\ &= \frac{1}{\sqrt{3}}\,[(\varepsilon_{11} - \varepsilon)^2 + (\varepsilon_{22} - \varepsilon)^2 + (\varepsilon_{33} - \varepsilon)^2 + 2\,(\varepsilon^2_{12} + \varepsilon^2_{23} + \varepsilon^2_{31})]^{1/2}\\ &= \frac{1}{3}\,[(\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2 + 6\,(\varepsilon^2_{12} + \varepsilon^2_{23} + \varepsilon^2_{31})]^{1/2}.\end{aligned} \qquad (16)$$

6. THE SHEARING STRESS INTENSITY

The component $D\,\mathbf{n} = D_2\mathbf{n}$, in the vector space of the principal shear stress, is given by

$$D\,\mathbf{n} = D_2\,\mathbf{n} = \sqrt{\frac{2}{3}}\ (v_1\,\mathbf{f}_1' + v_2\,\mathbf{f}_2' + v_3\,\mathbf{f}_3').$$

In this vector space, the vector $D\,\mathbf{n}$, ″is projected″ with enlargement, i.e.

$$\sqrt{\frac{3}{2}}\,D\,\mathbf{n} = \sqrt{\frac{3}{2}}\,D_2\,\mathbf{n} = v_1\,\mathbf{f}_1' + v_2\,\mathbf{f}_2' + v_3\,\mathbf{f}_3'. \tag{17}$$

The unit vectors parallel to principal shear stresses are given by

$$\mathbf{f}'_1 = \frac{\sqrt{2}}{2}\ (\mathbf{e}'_2 + \mathbf{e}'_3),\quad \mathbf{f}'_2 = \frac{\sqrt{2}}{2}\ (\mathbf{e}'_3 + \mathbf{e}'_1),\quad \mathrm{f}'_3 = \frac{\sqrt{2}}{2}\ (\mathbf{e}'_1 + \mathbf{e}'_2).$$

In ortohonormal base ($\mathbf{f}'_1$, $\mathbf{f}'_2$, $\mathbf{f}'_3$) the vector $\mathbf{n}$ is defined by

$$\mathbf{n} = (\mathbf{n}\cdot\mathbf{f}'_1)\,\mathbf{f}'_1 + (\mathbf{n}\cdot\mathbf{f}'_2)\,\mathbf{f}'_2 + (\mathbf{n}\cdot\mathbf{f}'_3)\,\mathbf{f}'_3 \tag{18}$$

where $\mathbf{n}\cdot\mathbf{f}'_1 = \mathbf{n}\cdot\mathbf{f}'_2 = \mathbf{n}\cdot\mathbf{f}'_3 = \frac{1}{\sqrt{3}}$. The vector $\sqrt{\frac{3}{2}}\ D_2\,\mathbf{n}$ lies on the plane of the unit normal $\mathbf{n}$, therefore is $\sqrt{\frac{3}{2}}\ D_2\,\mathbf{n}\cdot\mathbf{n} = 0$, and this given

$$v_1 + v_2 + v_3 = 0. \tag{19}$$

By (19) is determined the Hessian normal form the equation of the plane in the vector space of the principal shearing stresses. The plane (19) goes through the origin of the orthonormal base ($\mathbf{f}'_1$, $\mathbf{f}'_2$, $\mathbf{f}'_3$).

In the conditions of pure shearing, if $\varepsilon = 0$, $v_1 = v_2 = 0$, $v_3 = v_{max}$ [5], the strain vector $B\,\mathbf{n}$ is equal to the deviatoric components $D\,\mathbf{n} = D_2\,\mathbf{n}$ In this case is

$$\sqrt{\frac{3}{2}}\,B\,\mathbf{n} = \sqrt{\frac{3}{2}}\,D\,\mathbf{n} = \sqrt{\frac{3}{2}}\,D_2\,\mathbf{n} = v_3\,\mathbf{f}_3' \tag{20}$$

or

$$\sqrt{\frac{3}{2}}\,B\,\mathbf{n} = \sqrt{\frac{3}{2}}\,D\,\mathbf{n} = \sqrt{\frac{3}{2}}\,D_2\,\mathbf{n} = \tau_3\,\mathbf{f}_3' = \tau_{max}\,\mathbf{f}_3' = \tau_i\,\mathbf{f}_3'. \tag{21}$$

The quantity

$$\tau_i = \frac{\gamma_i}{2} = \sqrt{\frac{3}{2}}\ |D\,\mathbf{n}| = \sqrt{\bar{I}_2'}\,, \tag{22}$$

where $\bar{I}_2'$ is non-negative values of the second invariants of stress deviator, is called shearing stress intensity.

7. CONCLUSION

The normal strain intensity corresponds to the elastic energy of distortion apart from a constant factor. The normal strain intensity, sometimes, a reduced strain or strain intensity.

The shearing strain intensity is proportional to the quadratic invariant of the strain deviator, which may be regarded as an overall characteristic of the distortion of an element. The shearing strain intensity becomes zero only if the state of stress is a state of hydrostatic pressure. For pure shear in the plane v_{max} is the shearing stress intensity. In the case of the simple tension (compression) in the $\mathbf{e}'_1$ directions $\sqrt{3}|\varepsilon_i|$ is the shearing stress intensity $|\gamma_i|$.

8. REFERENCES

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