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THE INFLUENCE OF THE STIFFNESS AND THE THIRD BEARING ON MODAL PARAMETERS OF A BEAM

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ABSTRACT
The results of dynamic analysis of an axisymmetric engineering structure is shown in this paper. The influence of bearing stiffness and the third bearing on natural frequencies and mode shapes of a Timoshenko beam model was analyzed using the software Ideas Master Series 6. The results show that rising the natural frequencies of the system can be accomplished by proper choice of bearing stiffness and the third bearing location.
Key words: natural frequency, mode shape, beams, bearings

1. INTRODUCTION
Dynamic characteristics such as natural frequencies and mode shapes depend a lot of bearing stiffness and location. In many engineering applications, for instance in high precision manufacturing systems, the aim is to avoid a resonant state. In that sense, rising the natural frequencies of the system is of primary concern. Besides mounting the stiffer bearings, another possibility is to use the third bearing. The influence of the third bearing stiffness and location on natural frequencies as well as on mode shapes of vibrating axisymmetric structure was analysed numerically in this paper.

2. NATURAL FREQUENCIES AND MODE SHAPES OF A SIMPLY SUPPORTED BEAM
Dynamic characteristics of beams and shafts are greatly influenced by bearing stifness. Generally speaking, when bearing stiffness increases the natual frequencies of a beam also rise. But the bearing stiffness also influences the mode shapes of a beam.

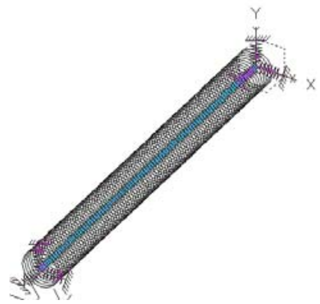

Figure 1. Numerical model of a simply supported beam

The numerical analysis was performed on a beam with diameter 0.1 m and 1m in length, [1]. The numerical model consisted of 100 equal Timoshenko beam elements, Figure 1. Bearings were simulated as linear translational stiffnesses in three directions. The bearing stiffnesses k_x and k_y were varied from 10^3-10^{15} N/m. Bearing stiffness in axial direction was kept constant $k_z=10^{15}$ N/m. The natural frequencies of two rigid body modes f_{R1} and f_{R2} and three bending modes f_{B1}, f_{B2}, f_{B3} are calculated using the software Ideas Master Series 6, Figure 2. The natural frequencies of the first torsional and axial modes were not influenced by change in k_x and k_y (constant values: f_T=1600,85 Hz and f_A=2571,34 Hz). The diagram shown in Fig.2 is known as Undamped Critical Speed Map. It can be seen that natural frequencies of bending mode vibrations change the most in the bearing stiffness range from 10^6 to 10^{10} N/m. Also, the mode shape tranformation happens in that range. This fact is important because the real bearing stiffness lies in that range, [2]. On the other side, natural frequencies and mode shapes remain almost constant for stiffnesses lower and higher from that range. Also the ratio between the first rigid body modes is constant and equal to $3^{1/2}$, which is the value published in literature [3].

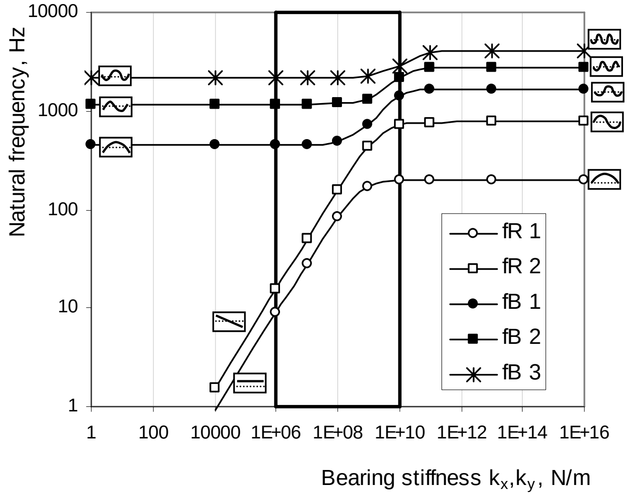

Figure 2. Natural frequencies and mode shapes versus bearing stiffness (two end bearings)

It is interesting that for axysymmetric structures with another ratio d/L, bounding values for the transferring range change a little. More accuratelly said, the range slowly moves to the right (higher stiffness) with increase in the ratio d/L.

3. THE INFLUENCE OF THE THIRD BEARING

3.1. The influence of bearing stiffness

It is common in engineering practice to support a shaft by three bearings. Often this is the case in machine tool spindles, where more stiff system gives better manufacturing accuracy. In that case the total stiffness of a shaft-bearing structure rises, and consequently the natural frequencies also increase.

To analyze the influence of the third bearing, the numerical analysis of a model shown in Fig.3 was performed. The third bearing had the same stiffness as the two at the beam ends, which simulate the case of three identical bearings. The influence of the third bearing stiffness on natural frequencies and mode shapes of the beam was analyzed for the case when the third bearing was mounted in the middle of the beam.

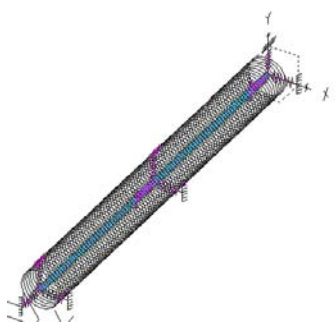

Figure 3. A beam supported by three bearings

The change of natural frequencies versus all three bearing stiffnesses is shown on Fig.4. The highest change in frequency due to inserting of the third bearing happens in the first rigid mode. This difference is small for lower bearing stiffness (up to 10^7 N/m), and then gradually rise to a new constant value for higher stiffnesses. The same is also obvious for the other modes with a note that this difference is negligible for low stiffnesses. In transition range from 10^7-10^8 N/m to 10^{12} N/m, mode shapes transform to more complex forms for higher modes. Mode shapes for the structure with three equally spaced bearings are also shown on Fig.4.

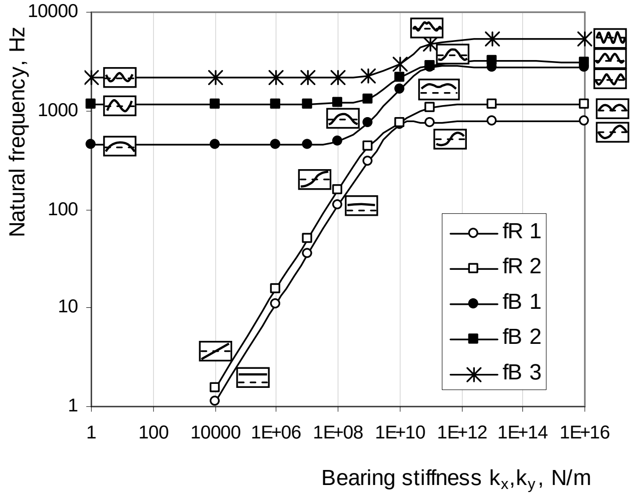

Figure 4. Natural frequencies and mode shapes versus bearing stiffness (three equally spaced bearings)

3.2. The influence of the third bearing location

The numerical analysis is also performed for the case when the third bearing moves from the left end of the beam to the right. The stiffnesses of all bearings were the same. The transversal stiffness was varied from 10^6 to 10^{11} N/m. The results are shown on Fig 5 and Fig.6.

The first torsional frequency was not changed (f_T=1600,85 Hz) beacause the third bearing linear stiffness didn′t influence that mode. The axial frequency increases to the value f_A=5143,267 Hz with the third bearing movement to the middle of the beam span (due to axial stiffness $k_z=10^{15}$ N/m).

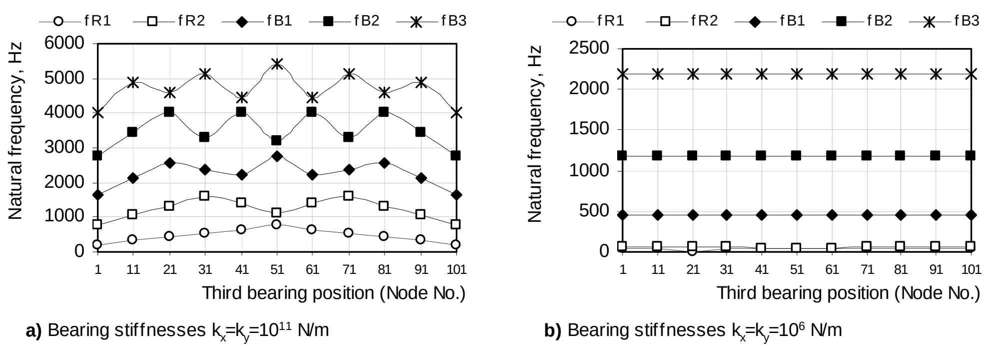

Figure 5. Natural frequencies change (two rigid and three bending modes) versus the third bearing position for: a) high and b) small bearing stiffnesses

The influence of the third bearing can be summarized as follows:

- For high stiffnesses (10^{10} and 10^{11} N/m), natural frequencies of previously rigid body modes (which are not rigid anymore) change a lot with the third bearing location, Fig.5a.
- For small stiffnesses (10^6 and 10^7 N/m), the third bearing location doesn′t influence much on the natural frequencies change, Fig.5b.

- The highest natural frequency of the first rigid body mode happens for the middle location of the third bearing, Fig.6a.
- The highest natural frequency of the second rigid body mode happens for the locations approximatelly on one third of the beam span., Fig.6b.
- The natural frequencies of the first three bending modes change a lot with the third bearing location. For equally spaced bearings, natural frequencies of these modes are maximal or minimal depending on the shape of relevant vibration mode, Figs.6c,d,e
- Mode shapes of these modes are greatly influenced by the third bearing location, Fig.4.

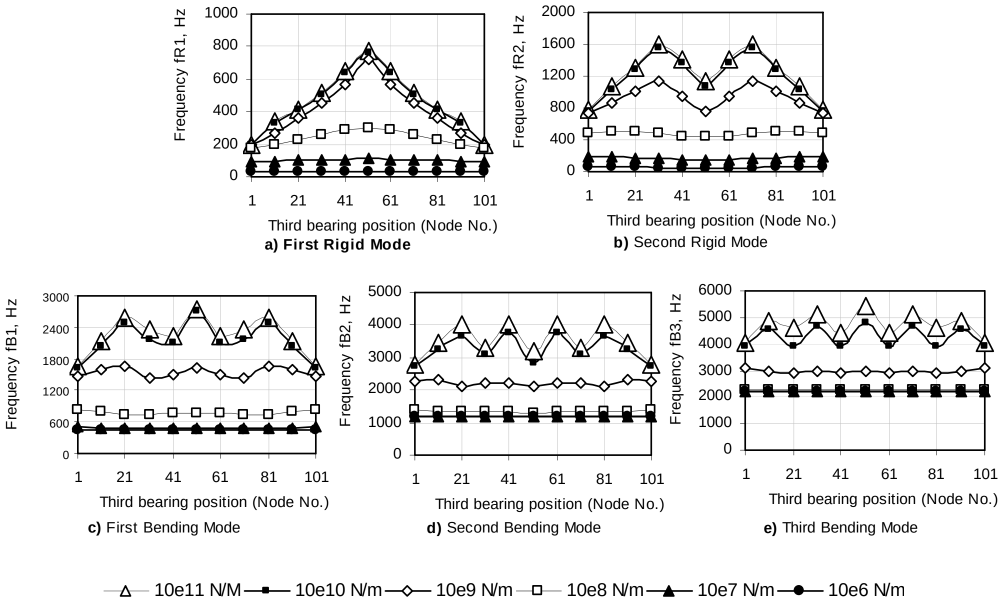

Figure 6. Frequencies of rigid body and bending modes versus third bearing position for different bearing stiffnesses

4. CONCLUSIONS

The dynamic characteristics of an axisymmetric engineering structure depends a lot of bearing stiffness and location. In many engineering applications, for instance in high precision manufacturing systems, the aim is to avoid a resonant state. In that sense, rising the natural frequencies of the system is of primary concern. Besides mounting the stiffer bearings, another possibility is to use the third bearing. The influence of the third bearing stiffness and location on natural frequencies as well as on mode shapes of vibrating axisymmetric structure was analysed numerically in this paper.

For the beam structure considered here, an optimal location for the third bearing would be on one third of the beam span because in that case the first natural frequency of rigid body mode rise, and the second reaches its maximum value. This isn′t valid for bearings with low stiffness (beyond 10^8 N/m) but this is not a real case in practice.

5. REFERENCES

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