

## COMPARISON OF SOME VIBRATION BASED DAMAGE IDENTIFICATION METHODS IN BEAMS

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### ABSTRACT

*The paper presents the results of four damage identification methods in beams. The first technique was developed by the author in [1] and presented in [2]. The comparison was conducted on the base of the available results presented in [3,4,5]. The identification of damage location and its depth in the first technique relies on previously established numerical model and regression relations between changes in natural frequencies and damage parameters. The other methods take into account changes in modal energy and stiffness variation with damage characteristics. Although the use of regression analysis makes the first method approximate, the results are quite satisfactory and at least as good as the results obtained by the other compared methods.*

**Key words:** vibrations, damage identification

### 1. INTRODUCTION

Vibration based damage identification techniques are based on the fact that physical properties of a structure are affected by presence of damage, which directly cause changes in modal parameters, such as natural frequencies, damping factors, and mode shapes of the structure. This means that monitoring the changes in modal parameters can be efficiently used for assessing the structural integrity, that is, to detect presence of a damage and its extent.

Various methods have been proposed to identify damage parameters in structures. The method of using vibration data for detecting cracks appeared in the 1940s. A comprehensive survey of the available literature can be found in [6,7]. These reports reviewed various methods on detection and identification of structural damage using vibration based testing.

This paper compares the results of damage identification presented in four published methods.

### 2. BRIEF SURVEY AND RESULTS OF THE COMPARED IDENTIFICATION METHODS

#### 2.1. Identification method No.1, [2]

The first identification technique was developed by the author in [1] and presented in [2]. Basic idea for this identification technique lies in the fact that natural frequencies of a structure depends on its mass and stiffness following the matrix equation for undamped natural vibrations, Eq.(1):

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \quad (1)$$

Here,  $\mathbf{M}$  is the mass matrix,  $\mathbf{K}$  is the stiffness matrix, and  $\ddot{\mathbf{q}}$  and  $\mathbf{q}$  are the acceleration and displacement vectors, respectively. The eigenvalues of Eq. (1) correspond to the undamped natural frequencies of the structure. Structural damages alter the mass and stiffness matrices and, consequently, the natural frequencies of a structure change.

The first technique refers to a simple case of a free-free beam, Fig.1. The numerical model of the beam had the following properties: length  $L_b=400$  mm, height  $H=8,16$  mm, width  $B=8,12$  mm, modulus of

elasticity  $E=206,8$  GPa, mass density  $\rho=7820$  kg/m<sup>3</sup>, and Poisson's coefficient  $\nu=0,29$ . The beam is modeled using solid elements in software I-DEAS Master Modeler 9. The damage is simulated as a

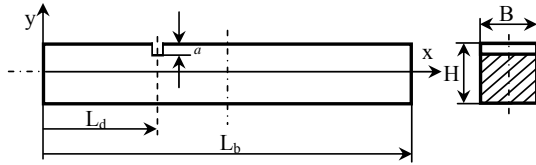


Figure 1. Geometry of the free-free beam with a notch

narrow open notch perpendicular to the beam axis. The location of the damage is  $L_d$ , its depth is  $a$ , and the width of the notch is 1mm. Relative location  $L = L_d / L_b$  and relative depth of the damage  $D=a/H$  were varied and the first four natural frequencies corresponding to the bending modes of the undamaged and the damaged beam were calculated. Due to structural symmetry, the location of the notch  $L_d$  measured from the left end of the beam was varied from 10

mm to 200 mm in 10 mm increments. The depth  $a$  of the notch was varied from 1 mm to 4 mm in 1 mm increments. Then, the relative frequency parameters  $F_1 = f_{i(d)} / f_{i(u)}$ ,  $i=1,2,3,4$ , were calculated. Here,  $f_i$  represents the  $i$ th natural frequency; subscripts (d) and (u) denote damaged and undamaged beams, respectively.

The numerical values of parameters  $F_1$ ,  $i=1,2,3,4$ , were taken as the input data for establishing the regression curves that describe relations between the relative frequency parameters and damage parameters (relative location and relative depth). The software STATISTICA 6.0 (Nonlinear Estimation option) was used for statistical estimation of these regression relations, [8]. The appropriate regression relations can be found by trial-and-error or intuitively for any type of the real beam taking into account its dimensions, material, boundary conditions, etc.

For the beam under consideration, the best results were obtained when regression relations include quadratic influence of the relative depth  $D$  and polynomial influence of the relative location  $L$ , as shown through Eqs. (2,3,4,5):

$$F_1 = 1 - 0.177566 D^2 (-0.01948 - 0.85975 L + 6.32585 L^2 + 47.5372 L^3 - 83.912 L^4) \quad (2)$$

$$F_2 = 1 - 0.42922 D^2 (0.065133 - 3.8651 L + 45.7407 L^2 - 44.275 L^3 - 267.41 L^4 + 406.144 L^5) \quad (3)$$

$$F_3 = 1 - 11.0353 D^2 (0.006469 - 0.37663 L + 5.74127 L^2 - 16.533 L^3 - 32.81 L^4 + 180.968 L^5 - 177.36 L^6) \quad (4)$$

$$F_4 = 1 - 69.938 D^2 (0.00252 - 0.17049 L + 3.30183 L^2 - 18.862 L^3 + 21.3852 L^4 + 121.863 L^5 - 375.07 L^6 + 298.339 L^7) \quad (5)$$

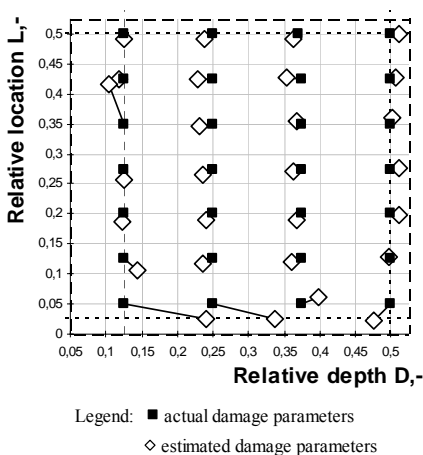


Figure 2. Results of damage identification method No.1

The corresponding coefficients of correlation are very high (0.996 for  $F_1$ ,  $F_2$ ,  $F_3$ , and 0.998 for  $F_4$ ).

The largest differences between the values obtained numerically and those calculated by regression relations are at those locations where nodes and maximal amplitudes of the particular mode shapes occur. This results from the nature of regression relations to smooth the data at extreme points.

Each of the regression surfaces  $F_1(D,L)$  when cut by a horizontal plane  $F_1 = \text{const.}$  gives the intersection curve which shows all possible values of  $D$  and  $L$  that refer to that frequency change. The intersection curves obtained by  $F_1$ ,  $i=1,2,3,4$ , should theoretically give a common intersection point in  $D-L$  plane, which shows the values of damage parameters  $D_{\text{est}}$  and  $L_{\text{est}}$ , see [9]. However, due to inevitable errors in modeling, regression analysis and measurement, it is

more reliable to locate this point using more frequency changes.

Using Eqs.(2)-(5), the values of the unknown  $L$  and  $D$  can be found for each combination of two different frequency parameters  $F_1$  and  $F_j$ ,  $I=J$  (excluding the complex values as well as negative values of  $D$ , which are meaningless).

This technique proposes finding three closest intersection points of frequency curves, i.e. those that give minimal sum of their distances from their mean value. The coordinates of their mean value can be adopted to represent the damage parameters. The only prerequisite here is that this mean value should not estimate the damage much beyond the numerically observed ranges of  $L$  and  $D$  (here, for  $L=0.025$  to  $0.5$  and  $D=0.125$  to  $0.5$ ) that are covered by regression relations. In such a case, the next combination of three intersection points giving minimal sum of distances from their mean value should be appropriate.

The procedure of finding three closest points was repeated for 28 cases of damage parameters (7 damage locations with 4 depths) using pseudo-experimental data, self-generated from the numerical model. The results obtained allow the evaluation of the procedure effectiveness and resolution. The results of this training examination are shown in Fig.2.

## 2.2. Identification method No.2, [3]

The identification method in [3] uses crack location model and crack size model that are formulated by relating fractional changes in modal energy to changes in natural frequencies due to damage. The verification of the proposed identification scheme is performed using the available test data for a free-free beam published in [10]. Test specimens were steel beams with 32 mm x 16 mm rectangular cross-section and 0,72 m long. The corresponding material properties were:  $E=206$  GPa,  $\nu=0,29$  and  $\rho=7650$  kg/m<sup>3</sup>. The results of this identification method using first four frequencies are shown in Fig.3.

Legend: ■ actual damage parameters ◇ estimated damage parameters

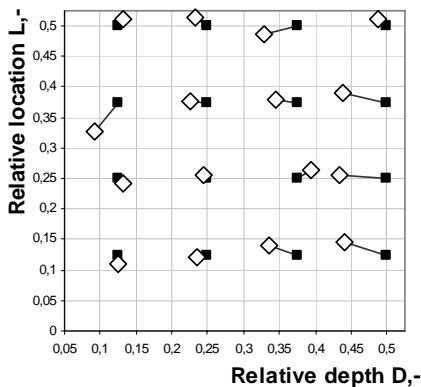


Figure 3. Results of identification method No.2

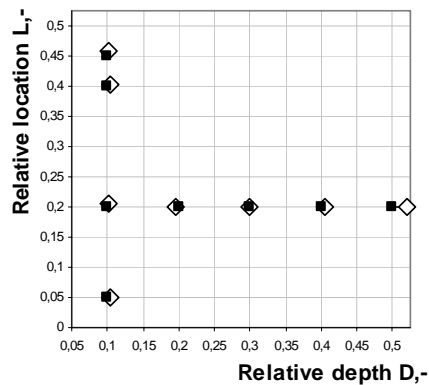


Figure 4. Results of identification method No.3

## 2.3. Identification method No.3, [4]

The method presented in [4] is based on finding the intersection points of plots representing the variation of stiffness with crack location. For analytical approach, the crack was represented as a rotational spring. The depth of the crack is then found using the fracture mechanics relation between stiffness and crack size. First three frequencies are used in this technique. For numerical analysis, the stepped beam was discretised by eight-noded isoparametric elements and quarter-point singularity elements were used around the crack tip. Material data were:  $E=210$  GPa,  $\nu=0,3$ ,  $\rho=7860$  kg/m<sup>3</sup>. The results published in [4] are shown in Fig.4.

## 2.4. Identification method No.4, [5]

Like in [4], the identification method presented in [5] is also based on finding the intersection points of curves representing the change of a structure stiffness with damage location. However, in this method

short Timoshenko beams are considered, which include the effects of shear deformation and rotational inertia. The crack extension is estimated from a change in the first natural frequency - using the fracture mechanics formulae which relate the stiffness and damage magnitude. The results of crack identification in a short cantilever beam are shown in Fig.5.

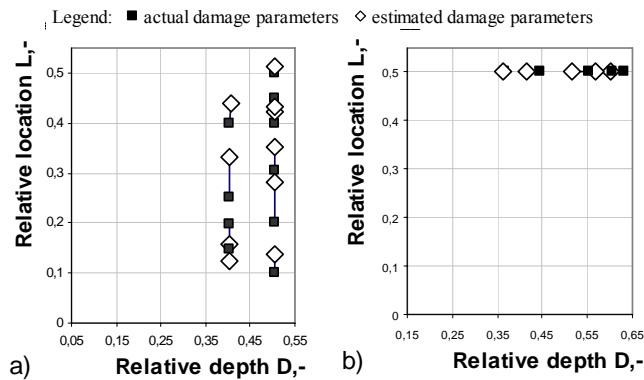


Figure 5. Results of damage identification method No.4:  
a) identification of relative location for  $D=0,405$  and  $D=0,506$ ;  
b) identification of relative depth for  $L=0,5$

### 3. CONCLUSIONS

The aim of the study was to show and compare the results of various damage identification methods based on vibration data. As can be seen, each of the compared methods is not absolutely accurate. Although the use of regression analysis makes method No.1 approximate, the results are quite satisfactory and at least as good as the results obtained by the other methods compared. The identification results are affected by modeling and measurements errors especially in case of small frequency changes due to minor cracks. Each technique requires different modeling efforts and computer time.

Consequently, great efforts are still put towards developing new, more reliable, efficient, and less tedious detection techniques. From an engineering point of view, the optimal method for damage identification should give as accurate results within reasonable time.

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