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AN ANALYSIS OF DAMPING TYPE INFLUENCE TO VIBRATION OF ELASTIC SYSTEMS

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ABSTRACT
In this paper an analysis of influence of three type of damping on free vibration of elastic systems is presented. Linear viscous, nonlinear viscous and dry friction damping were considered. Special emphasis was on determining of critical damping value for all of three types of considered damping cases. Also, analysis of logarithmic decrement of amplitudes, evaluation and comparison logarithmic decrement for considered damping types is presented. For all types of damping, same features of system were used and then analysed. Since some of types of damping couldn't be solved in closed form, numerical method was used. In this case, forth-order Runge-Kutta method is used for solving damping proportional to squared velocity of motion. Most important conclusion is that only system with linear viscous damping could have critical damping value.
Keywords: damping, critical damping, logarithmic decrement

1. INTRODUCTION
An often system that we could have in mechanical construction is spring-damper system. Different system requires different behavior of spring and damper. Most used kind of system is critically damped system. Critical damping is damping which represents border between harmonic and non-harmonic oscillations [1]. In critical damping, amplitude reach zero faster than any other oscillation.
Systems with linear damping have critical damping, and it could be determined with exact methods [2]. Logarithmic decrement of linear damped system is also constant. Besides linear damping, we also have damping proportional to squared velocity and dry friction. Real behavior of damping must be determined experimentally for concrete problems [3].
In this paper, critical damping of three types of damping would be analyzed. It is already known that linear critical damping could be determined by exact methods. Other two types of damping would be solved with numerical method by using fourth-order Runge-Kutta (RK). Since there is no exact solution, different properties (spring stiffness, damping coefficient, friction coefficient) would be considered and analyzed, and then we could make some conclusions. Logarithmic decrement would be analyzed for those three types of damping. Properties of those systems would be adjusted to obtain the same frequencies and initial displacement, and then analyzed and compared.

2. GOVERNING EQUATIONS

The simplest system used for this analysis is system with mass connected to a spring and damper, with or without friction.

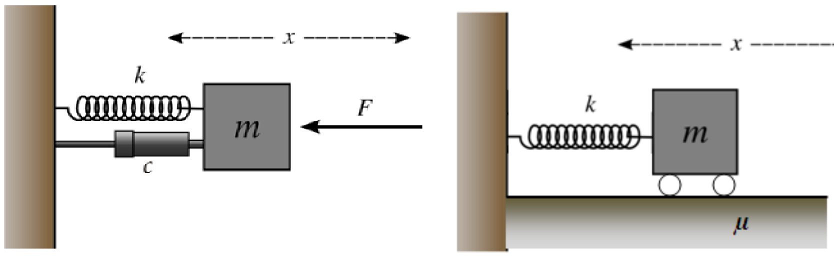

Figure 1. Mass-spring-damper system and mass-spring-dry friction system

Differencial equation of free damped oscilations could be written as [1]:

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0 \qquad (1)$$

where *c* – damping constant, *k* – spring constant, *m* – mass.
The same system with squared velocity have differential equation in following form [1]:

$$m\ddot{x} + c\dot{x}^2 + kx = 0 \qquad (2)$$

This differential equation could not be solved with exact methods. Solution for this equation would be found with fourth-order Runge-Kutta numerical method.
Third type of damping is system with dry friction. In differential equation of this damped system, member which represents damping is replaced with friction [4]:

$$m\ddot{x} + kx + sign(\dot{x}) \cdot \mu mg = 0 \qquad (3)$$

This differential equation has exact solution, which is compatible only when spring force and inertia force are larger than friction force. For this case, RK method would be used for solution.
Logarithmic decrement is natural logarithm of amplitude divided with following amplitude [5].

$$\lambda = ln\frac{a(t)}{a(t+T)} \qquad (4)$$

where *T* stands for period between two amplitudes.

3. ANALYSIS OF RESULTS

3.1 Analysis of three types of damping

First analyzed type of damping is linear damping. Linear damping has exact solution for critical damping in following form [1]:

$$x(t) = (A + Bt)e^{-\omega_0 t} \qquad (5)$$

Where *A* and *B* stands for:

$$A = x(0), \qquad B = \dot{x}(0) + \omega_0 x(0) \qquad (6)$$

and represents initial condition of damped system. We used $x(0) = 0.1$ and $\dot{x}(0) = 0$ as initial conditions, what is shown on the Fig. 2. This linear damping system is critical when condition $c = 2\sqrt{mk}$ is satisfied.

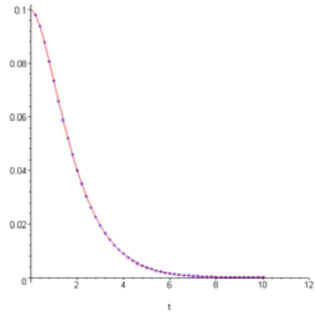

Figure 2. Critically damped linear system

Second analyzed type of damping is damping proportional to squared velocity. Solutions with different parameters of oscillations are shown on the Fig.3.

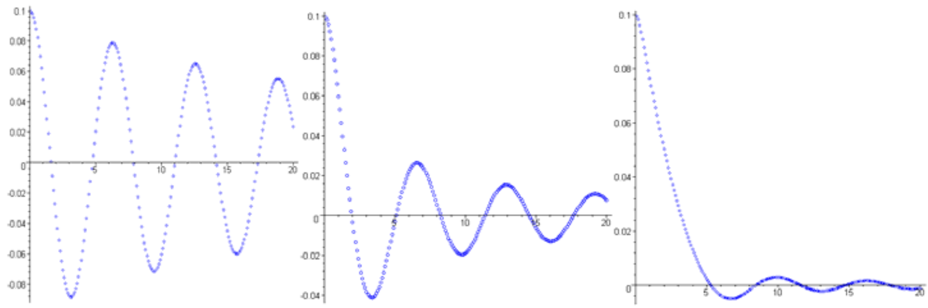

Figure 3. RK solution for damping with squared velocity

Third analyzed type of damping is system with dry friction. This system is solved with RK method, using different parameters for damping and friction constant. Solution is presented on the Fig. 4.

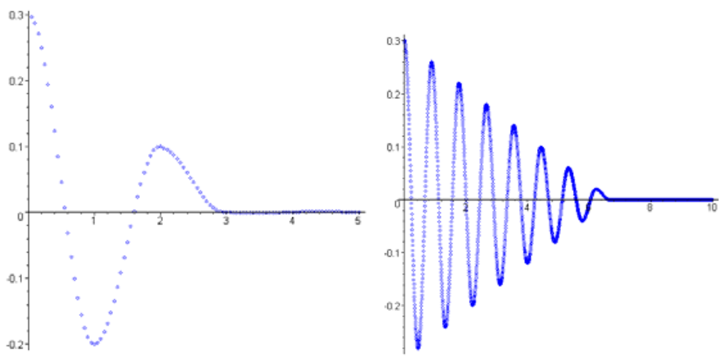

Figure 4. RK solution for system with friction

It is known that systems with linear damping have critical damping [1]. For other two types of damping we examined, we tried to figure their critical damping.

As we can see, on *figure 3*, even if we have large damping, we still have harmonic oscillations. Since critical damping is border between harmonic and non-harmonic oscillations, we can say that there is no critical damping in damping with squared velocity.

On *figure 4*, it is shown that friction also could not have critical damping. As soon as damping force becomes larger than spring force and inertia force, movement stops. Since mass that oscillates could never reach zero-position, critical damping is not possible in friction.

3.2 Analysis of logarithmic decrement

For analysis of logarithmic decrement, we used same initial conditions (displacement, velocity, frequency) for all three kinds of damped systems. Solutions are presented on the Fig. 5.

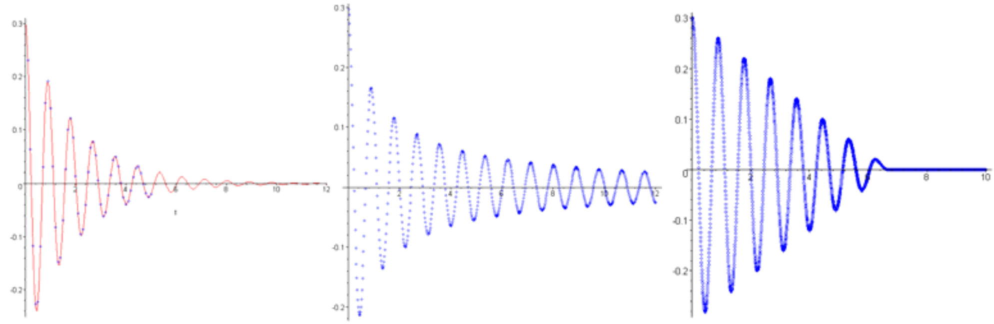

Figure 5. Linear damping, damping with squared velocity and friction

By measuring of oscilations amplitudes and calculating values and logarthmic decrement, results shown on the Fig. 6. could be obtained.

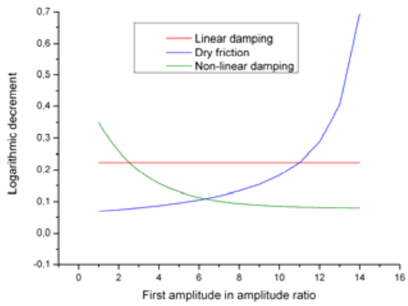

Figure 6. Logarithmic decrement for three types of damping

Logarithmic decrement of linear damping is also linear and constant. For other two types, logarithmic decrement is presented with exponential function (increasing for dry friction and decreasing for damping proportional to squared velocity).

4. CONCLUSION

In presented analysis of three types of damping, conclusion we could state is that only linear damping have critical value. Other two types of damping could not have critical damping value since it could not have non-harmonic oscilations (damping proportional to squared velocity) or it could never reach zero-point (dry friction). In mechanical systems, critical damping of linear damped systems have largest use in construction among all of analyzed three types of damping.

Linear damping have linear logarithmic decrement, and it is constant. Non-linear damping have exponecialy decreasing function of logarithmic decrement. Friction have constant difference between two following amplitudes, which makes logarithmic decrement an exponecialy increasing function.

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