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OPTIMIZATION OF PARAMETERS OF CYLINDRICAL GEAR PAIR

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ABSTRACT
In this paper are analyzed basic parameters of gear pair especially module, number of teeth and face width.
Optimization of parameters is done through objective function which is realized through the volume of cylindrical gear pair.
Optimization is realized through constrains which includes the safety factor of gear face and flank, gear ratio, ratio of face width to pitch diameter of pinion, module, number of teeth and face width of gears.
Keywords: optimization, gear pair optimization, gear transmitters.

1. INTRODUCTION
Gear power transmitters are part of mechanical group of high importance, which need to fulfill criterion for required performance.
Optimization of cylindrical gear pair is done through forming model of optimization which characterizes certain number of variables and objective function which in the concrete case is volume of cylindrical gear pair.
Optimization of cylindrical gear pair is important because based on the model established for gear pair optimization, we can create model of optimization for multi-stage gearboxes.
Analyses of forming model of optimization of cylindrical gear pair will be clarified through gear pair shown in figure 1, with the necessary data:

- Pinion material: steel, type Č.4732, for improvement,
- Gear material: steel, type Č.1731 for improvement,
- Standard module: m_{n12}=4.5 mm,
- Number of teeth of pinion: z_1=19,
- Number of teeth of gear: z_2=80,
- Face width: b_{12}=100 mm,
- Helix angle: $\beta_{12}=14^{o}$,
- Center distance a_{12}=230 mm,
- Torque of pinion: T_1=387.324 N· mm,

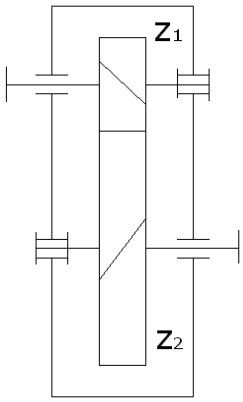

Figure 1. Scheme of cylindrical gear pair.

2. ENUNCIATION OF MODEL FOR CYLINDRICAL GEAR PAIR

For the concrete case of optimization as objective function is taken volume of cylindrical gear pair [1].

$$f(x) = \frac{\pi}{4} \cdot d_1^3 \cdot \varphi_I \cdot \left(1 + u_I^2\right) \tag{2.1}$$

If in the expression (2.1) we substitute expression for diameter d_1, ratio of face width to pitch diameter φ_I and gear ratio u_I, is obtained expression for objective function – volume of cylindrical gear pair.

$$f(x) = \frac{\pi}{4} \cdot \frac{m_n^2 \cdot b}{\cos^2 \beta} \cdot \left(z_1^2 + z_2^2\right) \tag{2.2}$$

Except this, in the model of optimization must be involved and constrains in the form of inequality from aspect of:

- Factor of safety from Pitting [5]:

$$S_H = \frac{[\sigma_H]}{\sigma_H}$$

Should satisfy the condition:

$$S_H \geq S_{H\min}$$

Factor of safety from Pitting has minimum value $S_{H\min} = 1.2$.

According to the inequality of the factor of safety for gear 1 and 2 are gained constraints in the form:

$$g_1(x) \equiv S_{H\min} - S_{H1} \leq 0$$
$$g_2(x) \equiv S_{H\min} - S_{H2} \leq 0$$

- Factor of safety from tooth breakage [5]:

$$S_F = \frac{[\sigma_F]}{\sigma_F}$$

Should satisfy the condition:

$$S_F \geq S_{F\min}$$

Factor of safety from tooth breakage has the minimum value $S_{F\min} = 1.4$.

According to the inequality of the factor of safety for gear 1 and 2 are gained constraints in the form:

$$g_3(x) \equiv S_{F\min} - S_{F1} \leq 0$$
$$g_4(x) \equiv S_{F\min} - S_{F2} \leq 0$$

- Gear ratio:
According to the inequality of gear ratio [2]:

$$u_{\min} \le u \le u_{\max}$$

Gained the constraints for gear ratio in the form:

$$g_5(x) \equiv u_{\min} - u \le 0$$

$$g_6(x) \equiv u - u_{\max} \le 0$$

Minimum gear ratio for gear pair is $u_{\min} = 4$, while the maximum gear ratio $u_{\max} = 4.4$.

- Ratio of face width to pitch diameter:

$$\varphi_{\min} \le \varphi \le \varphi_{\max}$$

Gained the constraints for ratio of face width to pitch diameter in the form:

$$g_7(x) \equiv \varphi_{\min} - \varphi \le 0$$

$$g_8(x) \equiv \varphi - \varphi_{\max} \le 0$$

Minimum ratio of coefficient is $\varphi_{\min} = 0.8$ while maximum is $\varphi_{\max} = 1.4$.

- Standard module:

$$m_{n\min} \le m_n \le m_{n\max}$$

Gained the constraints for standard module in the form:

$$g_9(x) \equiv m_{n\min} - m_n \le 0$$

$$g_{10}(x) \equiv m_n - m_{n\max} \le 0$$

Minimum standard module for gear pair is $m_{n\min} = 3\ mm$, while maximum is $m_{n\max} = 6\ mm$.

- Number of teeth [1]:

$$z_{\min} \le z \le z_{\max}$$

Are gained the constraints for numbers of teeth in the form:

$$g_{11}(x) \equiv z_{1\min} - z_1 \le 0$$

$$g_{12}(x) \equiv z_1 - z_{1\max} \le 0$$

$$g_{13}(x) \equiv z_{2\min} - z_2 \le 0$$

$$g_{14}(x) \equiv z_2 - z_{2\max} \le 0$$

Minimum number of teeth for gear pair are $z_{1\min} = 15$ and $z_{2\min} = 75$, while maximum $z_{1\max} = 25$ and $z_{2\max} = 85$.

- Face width:

$$b_{\min} \le b \le b_{\max}$$

Gained the constraints for face width in the form:

$$g_{15}(x) \equiv b_{\min} - b \le 0$$

$$g_{16}(x) \equiv b - b_{\max} \le 0$$

Minimum face width is $b_{\min} = 80\ mm$, while maximum is $b_{\max} = 110\ mm$.

Based on the objective function and functional constraints are identified variables of cylindrical gear pair, so that the design vector can be written in the form of relationship [3]:

$$x = x\left(m_{n12}, z_1, z_2, b\right) \tag{2.3}$$

From the results of program can be seen that the minimum value of objective function-volume of the cylindrical gear pair, is obtained for these optimal parameters of design vector [4].

$$x = \left(3; 19; 76; 80\right)$$

Value of objective function - volume of the cylindrical gear pair is:

$$f = 3.583764e + 0.06$$

In the figure 2 is represented minimization of objective function-volume of the cylindrical gear pair.

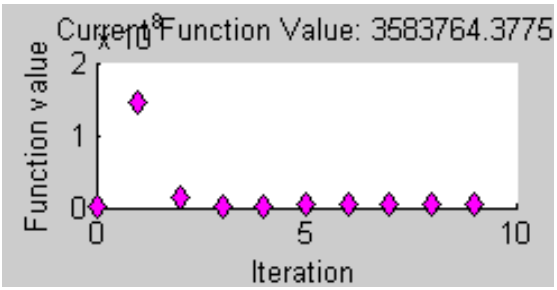

Figure 2. Minimization of volume of cylindrical gear pair.

From the figure 2 can be seen that after 9th iteration the objective function has its minimum value for optimal parameters.

3. CONCLUSION

According to data for cylindrical gear pair and optimization of parameters with Matlab program we can conclude that:

- Through the optimization of parameters are analyzed four parameters for each gear pair, are obtained their optimal values so that the objective function should be as small, and to satisfy all the set constraints.
- The obtained results of analyzed parameters represent the optimal values, based on which is obtained minimum volume of cylindrical gear pair.

4. REFERENCES

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