CONTENTS AUTHORS

15th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2011, Prague, Czech Republic, 12-18 September 2011

MATHEMATICAL MODELING OF FRICTION FORCE AND FRICTION COEFFICIENT AT DEEP DRAWING PROCESS

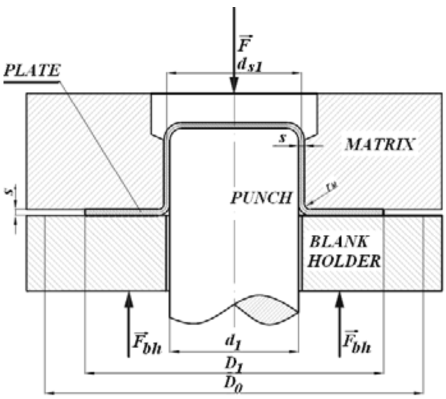
Prof. dr. Milan Jurković
Faculty of Technical Engineering, University of Bihać
Street Dr. Irfana Ljubijankića, Bihać
Bosnia and Herzegovina

Muslić Merima, dipl. ing.
Faculty of Technical Engineering, University of Bihać
Street Dr. Irfana Ljubijankića, Bihać
Bosnia and Herzegovina

ABSTRACT
This paper includes: experimental determination of process and blank holder forces at plate deep drawing, calculation of friction force and friction coefficient on the basis of achieved experimental results, determination of mathematical models $F_t = f(SHP, \varphi, p_d)$ and $\mu = f(SHP, \varphi, p_d)$, as well as graphical simulations and analyses of achieved mathematical models.
As substance for cooling and lubrication molybdenum disulfide and oil EP50 were used, while tribological conditions of contact surfaces were varied using phosphated and non-phosphated plate (EN 10027-1: DC04; DIN: RRSt14). Research results have shown that the best tribological conditions were achieved with molybdenum disulfide on phosphated plate, and the most influencing parameter is material deformation degree.
Keywords: deep drawing, tribology, process force, blank holder force, friction force, coefficient of friction, mathematical modeling, simulation and optimization of models

1. EKSPERIMENTAL DETERMINATION OF PROCESS AND BLANK HOLDER FORCE

In the master paper, study of the tribological condition of deep drawing process of cold rolled steel plate (EN 10027-1: DC04; DIN: RRST14) with constant thickness of 3 mm, was carried out. Schematic process description is shown on the figure 1 as follows:



where:
d_1 - Product internal diameter,
d_{s1} - Product medial diameter,
D_0 - Initial blank diameter,
D_1 - Wreath diameter,
s - Plate thickness,
F - Process force,
F_{bh} - Blank holder force.

Figure 1. Schematic of studied deep drawing process

As it was not possible to directly measure friction coefficient and friction force, therefore it was concluded about mentioned values on the basis of measured normal process forces and blank holder forces. [1,2]. Measuring of these forces was carried out using tensiometric tapes glued to the elastic sensor (Fig.2) from which signals where transmitted to 8 channel analog-digital converter "Spider 8" that was connected to PC (Fig. 3). With the help of "Catman 3" software obtained data are shown in table and graph [7].

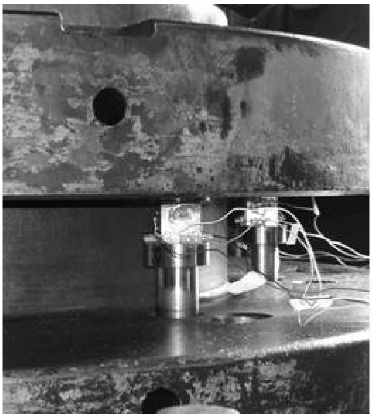


Figure 2. View of the sensors

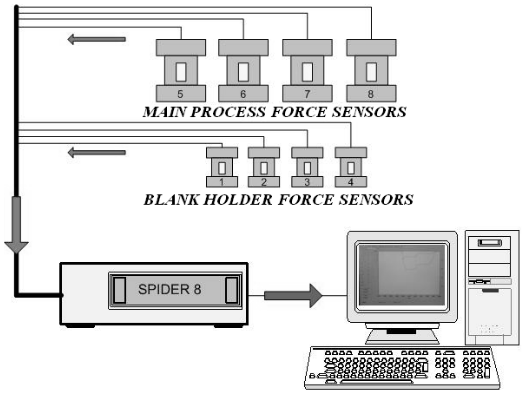


Figure 3. Funtional mesuring sheme

2. CALCULATION OF THE FRICTION COEFFICIENT

After measurements where completed average values of the process and blank holder forces are adopted. According to figure 1, and due to the press construction these forces are directly opposed to each other. With the help of the software drawing force F_i was achieved that according to [1,6,8] consists of the following components:

$$F_i = F_{pd} + F_s + F_{t1} + F_{t2} \tag{1}$$

F_{pd} – ideal forming force, or deep drawing plastic deformation force that is necessary for tangential forming on the rim, F_s– bending force on rounded tool (matrix) edge, F_{t1}– friction force in the flat rim area, F_{t2}– friction force of the rounded tool (matrix) edge.

Using generally accepted formulas [6,7,8] for calculation of mentioned components of the drawing force (F_{pd} i F_s) friction force and friction coefficient where determined for each test according to formulas:

$$F_t = F_i - F_{pd} - F_s \tag{2}$$

and

$$\mu = \frac{-2F_{bh} - 1{,}6F_{pd} + \sqrt{(2F_{bh} + 1{,}6F_{pd})^2 - 12{,}8F_{bh}(F_{pd} + F_s - F_i)}}{6{,}4F_{bh}} \tag{3}$$

Obtained results are shown in the table 2 in accordance with experiment plan – experiment matrix.

3. MATHEMATICAL MODELING OF THE FRICTION FORCE AND COEFFICIENT

When planning experiment mathematical model of higher range is assumed, that matches central composition plan shown in the figure 4. [3]. It is a three factor experiment plan that varies in three basic level (-1, 0,-1), with appropriate number of repetitions in the middle stage and additional tests in points–α and + α which are laid symmetrically to the center of plan.

Following variables where accepted when experiment was performed:

- Drawing depth h, or deformation degree $\varphi = lnD_0/d_{s1}$,
- Blank holder force F_{bh}, managed by change of the holder pressure p_d, and
- Tribological conditions – lubricant λ [3, 4, 5].

Mentioned factors are varied in five levels, where two different lubricants where used: 1-hydraulic oil EP50, viscosity ν=50 [mm^2/s] and 2-molibden disulfide. To achieve five levels with different tribological conditions, phosphated and non-phosphated plates where used.

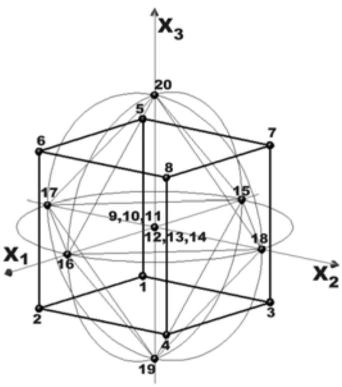


Figure 4. Central composition plan

Table 1. Variation levels of the influencing factors and coded values

Coded values	X_i	-α	-1	0	+1	+ α
Physical values	λ	λ_{FM}* =0,16	λ_{M}=0,5	λ_{FU}=1	λ_{U}=1,5	λ_{F}=1,84
	h / φ	42/0,305	50/0,321	62/0,341	74/0,361	82/0,375
	p_d [bar]	40	44	50	56	60

*FM – phosphated plate + molybdenum disulfide, M – non-phosphated plate + molybdenum disulfide, FU – phosphated plate + oil, U – non-phosphated plate +oil, F – dry phosphated plate with no lubricant.

Influencing factors and its varying levels are shown in the Table 1. Variable λ is introduced to describe in quantity lubrication factor [3,4,5].

Table 2. Experiment matrix plan

N	Coded values											Physical Values			Values calculated according to (2) and (3)	
	X_0	X_1	X_2	X_3	X_1X_2	X_2X_3	X_1X_3	$X_1X_2X_3$	X_1^2	X_2^2	X_3^2	λ	φ	p_d (bar)	F_t [kN]	μ
1	+1	-1	-1	-1	+1	+1	+1	-1	+1	+1	+1	0,5	0,321	44	142,26	0,067
2	+1	+1	-1	-1	-1	+1	-1	+1	+1	+1	+1	1,5	0,321	44	267,26	0,120
3	+1	-1	+1	-1	-1	-1	+1	+1	+1	+1	+1	0,5	0,361	44	222,19	0,095
4	+1	+1	+1	-1	+1	-1	-1	-1	+1	+1	+1	1,5	0361	44	287,19	0,124
5	+1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	0,5	0,321	56	177,26	0,072
6	+1	+1	-1	+1	-1	-1	+1	-1	+1	+1	+1	1,5	0,321	56	272,26	0,110
7	+1	-1	+1	+1	-1	+1	-1	-1	+1	+1	+1	0,5	0,361	56	242,19	0,093
8	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	1,5	0,361	56	322,19	0,121
9	+1	0	0	0	0	0	0	0	0	0	0	1	0,341	50	214,97	0,089
10	+1	0	0	0	0	0	0	0	0	0	0	1	0,341	50	214,97	0,089
11	+1	0	0	0	0	0	0	0	0	0	0	1	0,341	50	224,97	0,092
12	+1	0	0	0	0	0	0	0	0	0	0	1	0,341	50	199,97	0,083
13	+1	0	0	0	0	0	0	0	0	0	0	1	0,341	50	204,97	0,085
14	+1	0	0	0	0	0	0	0	0	0	0	1	0,341	50	209,97	0,087
15	+1	-α	0	0	0	0	0	0	α^2	0	0	0,16	0,341	50	119,97	0,047
16	+1	+α	0	0	0	0	0	0	α^2	0	0	1,84	0,341	50	432,69	0,136
17	+1	0	-α	0	0	0	0	0	0	α^2	0	1	0,305	50	94,75	0,043
18	+1	0	+α	0	0	0	0	0	0	α^2	0	1	0,375	50	318,46	0,123
19	+1	0	0	-α	0	0	0	0	0	0	α^2	1	0,341	40	209,97	0,102
20	+1	0	0	+α	0	0	0	0	0	0	α^2	1	0,341	60	239,97	0,088
Assumed model: $Y_i = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{23}X_2X_3 + b_{13}X_1X_3 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2$																

Regression coefficients for assumed mathematical models are calculated independently from each other on the basis of the system of equations [3]:

$$b_0 = a_1 \sum_{j=1}^{N} Y_j + a_2 \sum_{i=1}^{k} \sum_{j=1}^{N} X_{ij}^{\ 2} Y_j \qquad i = 0, 1,2, ... k \qquad (4)$$

$$b_i = a_3 \sum_{j=1}^{N} X_{ij} Y_j \qquad i = 0, 1,2, ... k \qquad (5)$$

$$b_{im} = a_4 \sum_{j=1}^{N} X_{ij} X_{mj} Y_j \qquad 1 \leq i \leq m \leq k \qquad (6)$$

$$b_{ii} = a_5 \sum_{j=1}^{N} X_{ij}^{\ 2} Y_j + a_6 \sum_{i=1}^{k} \sum_{j=1}^{N} X_{ij}^{\ 2} Y_j + a_7 \sum_{j=1}^{N} Y_j \qquad (7)$$

Where following data where read out for assumed mathematical model according to [3]: α = 1,682; n_0 = 6; N = 20; a_1 = a_{11}= 0,1663; a_2 = a_7 = a_{17}= -0,0567; a_3 = a_{12}= 0,0732; a_4 = a_{13}= 0,125; a_5= 0,0622; a_6= a_{18}= 0,0068; a_{14}= 0,0693.; X_{ij} – coded values and Y_{ij} – values of the friction force and coefficient according to Table 2. After mathematical analysis which included: testing significance of regression coefficients, testing the adequacy and reliability of the model, calculating the coefficients of multiple regression and model decoding, following models have been adopted as a representative for the force and friction coefficient:

$$F_t = 89{,}12\lambda^2 - 47{,}8\lambda + 2163\varphi + 1{,}7749p_d - 655{,}46 \qquad (8)$$

$$\mu = 0{,}0123\lambda^2 + 0{,}0001207p_d^{\ 2} + 0{,}16131\lambda + 1{,}1406\varphi - 0{,}01207p_d - 0{,}4173\lambda\varphi - 0{,}0314 \qquad (9)$$

6. CONCLUSION

Simulation and analyses of mathematical models (8) and (9) leads to the conclusion that for the studied deep drawing process the best tribological conditions were achieved using molybdenum disulfide as lubricant on phosphated plate. At these conditions minimal values of the friction force and coefficient were appeared. Most unfavorable tribological conditions were during so called “dry” deep drawing process, as expected. When choosing between molybdenum disulfide and oil, it is better to use molybdenum disulfide since it reduces process force from 8-10% to maximally 30%, depending from other factors. Deformation degree as well has shown as influencing factor where for smaller changes in its values friction force and factor are rapidly increased.

7. REFERENCES

[1] Evin E., Wessely E.: Application of numerical methods by simulation deep drawing process, 6th International DAAM Symposium, Technical University of Krakow, Poland, October 1995.

[2] Hu Z., Vollertsen F.: A new friction test method, Journal for Technology of Plasticity, Vol. 29, 2004.

[3] Jurkovic M.: Matematicko modeliranje inzenjerskih procesa i sistema, Masinski fakultet – Bihac, 1999.

[4] Jurkovic M., Jurkovic Z., Buljan S.: The tribological state test in metal forming processes using experiment and modelling, Journal of Achievements in Materials and Manufacturing Engineering, Vol.18, 2006.

[5] Jurkovic Z., Jurkovic M., Kuzman K.: Experimental friction test by sheet metal deep drawing process, 7th International Conference on Technology of Plasticity ICTP, Yokohama, Japan, 2002.

[6] Musafia B.: Obrada metala plasticnom deformacijom, Svjetlost Sarajevo, 1991.

[7] Pašalić S.: Prilog istrazivanju utjecaja sile drzaca lima na kvalitet i stabilnost procesa dubokog izvlacenja, Magistarski rad, Univerzitet u Bihacu, Tehnicki fakultet Bihac, januar 2009.

[8] Stefanovic M.: Tribologija dubokog izvlacenja, Jugoslovensko Društvo za Tribologiju, Kragujevac, 1994.