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ANALYSIS OF THE MULTI-MASS DRIVE SYSTEM DYNAMICS WITH INDUCTION MOTOR

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ABSTRACT
The objective of this paper consists in mathematical modelling of the multi-mass drive system with induction motor. Transient performance of any electrical drives is greatly affected by each element of drive. The motor is described through d, q- axis model which is universally acceptable to investigate transient processes. MATLAB/SIMULINK based modelling is adopted to calculate the transient performance of three-phase induction. The developed model can be used to investigate the transient process with variations of parameters of entire electrical drive with induction motor.
Keywords: Electric drives with induction motor, mathematic modeling.

1. INTRODUCTION
Advanced electric drive is a mechatronic system, which contains both mechanical and electronic units. Induction drives operate at transients when the amplitude and frequency of electric variables or speed vary in time. The typical examples of induction motor transients are: direct staring after turn off, sudden mechanical loading sudden short circuit, reconnection after a short supply fault, and behaviour during short intervals of supply voltage reduction, performance with PWM (pulse width modulation) converter fed, etc. In the modern drive system a precise motion control is required. The imperfections of the mechanical system, such as the finite stiffness of the shaft, friction and backlash, can affect the dynamical characteristics significantly. For the classical control strategies all above mentioned factors make the control problem very difficult. For the modern systems the simplification of the drive to a one-mass model leads to unsatisfactory control results. This paper presents a comprehensive model of electrical drives with induction machine. An accurate and systematic d-q model of the induction machine is made in Matlab/Simulink environment. The model is a detailed transient electrical model [1] based on the conventional two-axis lumped parameter d-q equivalent circuit of the induction machine. In the paper the modelling of the multi-mass system simplification to the transient operation of the drive was presented.

2. THE COMPLEX MATHEMATICAL MODEL OF ELECTRICAL DRIVES WITH AC INDUCTION MOTOR

2.1. Modeling of AC motor
There are made several assumptions to simplify thinking over the three-phase induction motor [1]: the three-phase motor is symmetrical; only a basic harmonics is taking in to account, the spatially distributed stator and rotor windings are replaced by a concentrated coil, an anisotropy effects, magnetic saturation, iron loses and eddy currents are not taking into considerations, the coil resistance's and reactance's are taking to be constant, in many cases, especially when considering steady states, the currents and voltages are taking to be sinusoidal. Taken in consideration above

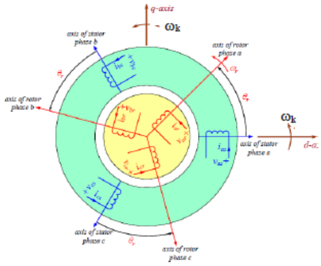


Figure 1. Schematic presentation of three phase induction motor, in ab,c coordinates and d,q,o.

assumptions the induction motor can be observed as a system of electric and magnetic circuits, which are coupled magnetically and electrically. Therefore the three phase induction motor is represented by six circuits (one per phase). Each of them includes self inductance and 5 mutual inductances. The stator and rotor inductances as well as mutual inductances between stator or rotor phases do not depend on rotor position. The mutual inductance between stator and rotor phases depends on rotor position. So the 8th order nonlinear model with time variable coefficients describes the transients in the induction motor. To reduce number of equations, determining performance of induction motor, the complex variable model is used, where the complex variables of stator and rotor currents are introduced as space phasors [5, 6]. As The use of space vectors as complex state variables is an efficient method for ac machine modelling. To describe the space vector concept, a three-phase stator winding is considered with the respective three-phase currents i_{as}, i_{bs} and $i_{cs,}$ Fig. 1. The resulting equation:

$$\vec{i}_s = \frac{2}{3}(i_{as} + a i_{bs} + a^2 i_{cs}) \qquad (1)$$

with $a=e^{j2\pi/3}$ defines the complex stator current space vector. In the same way as the phase currents define the stator current space vector, the phase voltages at the machine terminals define the stator voltage space vector

$$\vec{u}_s^s = R_s \vec{i}_s + \frac{d\vec{\Psi}_s}{dt} \cdots \qquad \cdots \vec{u}_r^r = R_r \vec{i}_r + \frac{d\vec{\Psi}_r}{dt} \qquad (2a,b)$$

where $\vec{u}_s^s$ and $\vec{u}_r^r$ and are voltage space phasors; $\vec{\Psi}_s$ and $\vec{\Psi}_r$ are phasor flux linkages of stator and rotor, expressed as:

$$\vec{\Psi}_s^s = L_s \vec{i}_s + Msr \cdot \vec{i}_r \cdot e^{j\theta_{sr}} \cdots \qquad \cdots\cdots \vec{\Psi}_r^r = L_r \vec{i}_s + Msr \cdot \vec{i}_s \cdot e^{j\theta_{sr}} \qquad (3)$$

and:

$$L_s = L_s + M_{sr} \cdots \qquad \cdots\cdots\cdots\cdots\cdots L_r = L_r + M_{sr} \qquad (4)$$

where rotor position $\theta_{sr}= p_1\ \theta_{r;}$ p_1 is number of pole pairs, $\theta_{r;}$ is actual rotor position, L_s is inductance of stator, composed from self inductance $L_{s\sigma}$ and mutual inductance of stator and rotor windings M_{sr}, L_r is inductance of rotor, $L_{r\sigma}$ is rotor self inductance. The stator variables in Eq. (2a) are presented in stator coordinates, in equation (2b) – in rotor coordinates. Using rotation of stator and rotor coordinates at speed ω_k, the equations of flux linkages obtain the form:

$$\vec{u}_s = R_s \vec{i}_s + \frac{d\vec{\Psi}_s}{dt} + j\omega_k \vec{\Psi}_s \cdots \qquad \cdots \vec{u}_r = R_r \vec{i}_r + \frac{d\vec{\Psi}_r}{dt} + j(\omega_k - \omega_r)\vec{\Psi}_r \qquad (5)$$

R_s, R_r are phase resistances of stator and rotor and ω_r is speed of rotor. The complex variables (state phasors) can be decomposed in plane along two orthogonal axes, rotating with speed $\omega_{k,}$ [2]. In induction machine, the d-axis is assumed to align on a-axis at *t* = 0 and rotate with synchronous speed $\omega_{k.}$ The result of this transformation is that all time-varying inductances in the voltage equations of an induction machine due to electric circuits in relative motion can be eliminated, in this way will the voltage balance equation for the d, q coils in arbitrary reference frames, Fig.2, are as follows:

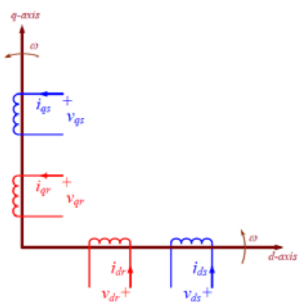


Figure 2. d,q,o, transformation

$$\begin{aligned}
U_{ds} &= \frac{d}{dt}\Psi_{ds} + \frac{R_S}{\delta \cdot L_S}\cdot\Psi_{ds} - \omega_k \cdot \Psi_{qs} - \frac{R_S \cdot L_m}{\delta \cdot L_S \cdot L_r}\cdot\Psi_{dr} \\
U_{qs} &= \frac{d}{dt}\Psi_{qs} + \frac{R_S}{\delta \cdot L_S}\cdot\Psi_{qs} - \omega_k \cdot \Psi_{ds} - \frac{R_S \cdot L_m}{\delta \cdot L_S \cdot L_r}\cdot\Psi_{qr} \\
U_{dr} &= \frac{d}{dt}\Psi_{dr} + \frac{R_r}{\delta \cdot L_r}\cdot\Psi_{dr} - (\omega_k - \omega)\Psi_{qr} - \frac{R_S \cdot L_m}{\delta \cdot L_S \cdot L_r}\cdot\Psi_{ds} \\
U_{qr} &= \frac{d}{dt}\Psi_{qr} + \frac{R_r}{\delta \cdot L_r}\cdot\Psi_{qr} - (\omega_k - \omega)\Psi_{dr} - \frac{R_S \cdot L_m}{\delta \cdot L_S \cdot L_r}\cdot\Psi_{qs}
\end{aligned} \qquad (6)$$

The electromagnetic torque is:

$$m_e = \frac{3}{2} p \frac{L_m}{\delta L_s L_r} (\psi_{qs}\psi_{dr} - \psi_{ds}\psi_{qr}) \tag{7}$$

2.2. Mathematical model of the mechanical system

The imperfections of the mechanical system, Fig.3, such as the finite stiffness of the shaft, friction and backlash, can affect the dynamical characteristics significantly. For the classical control strategies all abovementioned factors make the control problem very difficult. For the modern systems the simplification of the drive to a one-mass model leads to unsatisfactory control results. The major factor which affects the drive performances is the finite stiffness of the shaft. The speed control of the multi-mass system originally derives from the rolling mill drive [2].

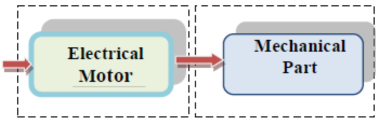


Figure3. The structure of an electric drive system

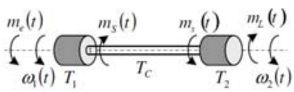
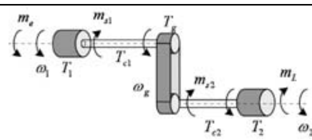
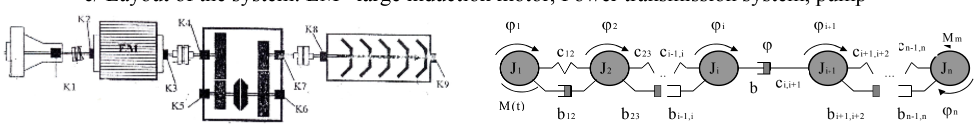
<table>
<tr><td>a/ The schematic diagram of the two-mass system with equations</td><td>b/ The schematic diagram of the three-mass system</td></tr>
<tr><td></td><td></td></tr>
<tr><td>A two-mass inertia-shaft-free model is described by the following equations [p.u.]:</td><td>A three-mass inertia-free-shaft system is depicted by the following equations [p.u]:</td></tr>
<tr><td>$$T_1 \frac{d\omega_1}{dt} = m_e - m_s;$$ $$T_2 \frac{d\omega_2}{dt} = m_s - m_L;$$ $$T_c \frac{d\omega_s}{dt} = \omega_1 - \omega_2;$$</td><td>$$T_1 \frac{d\omega_1}{dt} = m_e - m_{s1}; \cdots T_2 \frac{d\omega_2}{dt} = m_{s2} - m_L;$$ $$T_g \frac{d\omega_g}{dt} = m_{s1} - m_{s2}; \cdots T_{c1} \frac{d\omega_{s1}}{dt} = \omega_1 - \omega_g;$$ $$T_{c2} \frac{d\omega_{s2}}{dt} = \omega_g - \omega_2$$</td></tr>
<tr><td>where: ω_1 – motor speed, ω_2 – load speed, m_e – electromagnetic torque, m_s – shaft torque, m_L – load torque, T_1 – mechanical time constant of the motor, T_2 – mechanical time constant of the load machine, T_c – stiffness time constant.</td><td>where: ω_g – gearbox speed, m_{s1}- shaft torque in the beam between the motor and the gearbox, m_{s2} –shaft torque between the gearbox and the load machine, T_g- mechanical time constant of the gearbox.</td></tr>
<tr><td colspan="2">c/ Layout of the system: EM –large induction motor, Power transmission system, pump

c- stiffness coefficients; b- dumping coefficients; φ-angle of rotation</td></tr>
<tr><td colspan="2">The mechanical system will be treated with n-stage coordinates and based on equations of Lagrange-it, o second order : $$\frac{d}{dt}\left(\frac{\partial W_k}{\partial \dot{q}_r}\right) - \frac{\partial W_k}{\partial q_r} + \frac{\partial W_p}{\partial q_r} + \frac{\partial \phi}{\partial \dot{q}_r} = Q_r, \quad (r=1,2,3,..n).$$ where $L = Wk – Wp$ – Lagrange's function; Qi – force, depending on elementary works of external forces and their possible displacement ∂qi.</td></tr>
</table>

Figure 4. The schematic diagram of the a/ two b/ three and c/ multi -mass system

Large inertia of the motor and the rolls connected through a long shaft can tend to the torsional vibrations. In the analysis of the drive systems with flexible couplings the simplified mathematical models of the drive are commonly used. In the case of the inertia-shaft-free model of the drive the elastic coupling has the following parameters: the stiffness coefficient and the internal damping coefficient (which is very often neglected in typical industrial application). The existing inertia of the shaft is divided into two parts and added to the main system inertias (the motor and the load machine). In the analysis of the drive system with flexible coupling the following models are commonly used, [2]: the model with distributed parameters; the Rayleigh model, the chain model, the inertia-shaft-free model. The selection of a suitable model is a compromise between the obtained modeling accuracy and the calculation complexity. In the Fig. 4 are presented the mechanical system with two, three and multi masses system.

3. SIMULATION RESULTS

The developed mathematical model permits to simulate the induction motor starting process and takes in variable parameters of entire electrcial drive. In the Fig. 5 developed model, equations (1-7, with the mathematical model of mechanical system given in Fig. 4 will form mathematical model of and electrical drives with multi-mass system. This model can be used for both transients and steady-state analysis, and laso can include the variation of parameters of electrical machines and mechanical system.

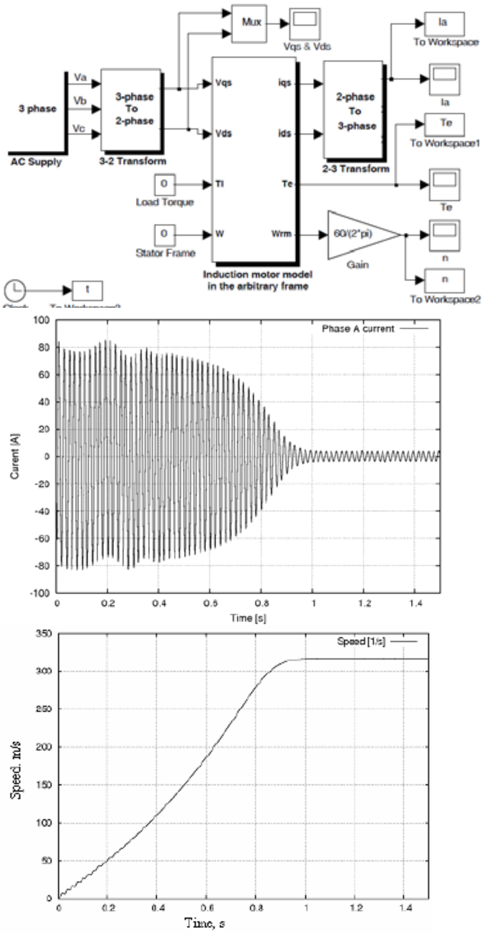


Figure 5. Typical Matab/Simulink blok used for simulation of electrical drive with induction motor. Parameters of modelled electrical drives with induction motor: U=220 V, 50 Hz, P=5.5 kW, n=300 rpm/min, R_s=1.05 Ω, L_s=3.6 mH, M_{sr}=253 mH; R_r=0.754 Ω, L_r=mH, J_r=0.0075 kgm^2.

4. CONSLUSION

The paper consists in mathematical modelling of the multi-mass drive system with induction motor. Transient performance of any electrical drives is greatly affected by each element of drive. The motor is described through d, q- axis model which is universally acceptable to investigate transient processes. MATLAB/SIMULINK based modelling is adopted to calculate the transient performance of three-phase induction. The developed model is used to investigate the transient process of electrical drive with induction motor.

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