15th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2011, Prague, Czech Republic, 12-18 September 2011

COMB-BASED METHOD FOR NARROWBAND FIR FILTER DESIGN

Gordana Jovanovic Dolecek
Department for Electronics, INAOE
E. Erro 1, 72840 Tonantzintla, Puebla
Mexico

Vlatko Dolecek
Faculty of Mechanical Engineering
Vilsonovo setaliste, b.b, 71000 Sarajevo
Bosnia & Herzegovina

Isak Karabegovic
Technical faculty
University of Bihac, Bihac
Bosnia & Herzegovina

Javier Diaz-Carmona
Technological Institute of Celaya
Celaya, 38010 GTO
Mexico

ABSTRACT
FIR (Finite impulse response) digital filters are known to have some very desirable properties such as linear phase, stability, and absence of limit cycle. However their applications generally require more computation. In this paper we consider the simple method to design a multiplier-less narrowband FIR filter. The method is based on well known IFIR (Interpolated FIR) structure. The novelty of the proposed approach is the new structure, where the interpolation filter is the cascade of comb filters and the wideband compensators. The comb filters and compensators are multiplier-less filters. The model filter is rounded to the nearest integers, which can be implemented with the adders and shifts. Consequently, the overall structure exhibits a low complexity and does not require the multipliers.
Keywords: FIR filter, comb filter, IFIR filter.

1. INTRODUCTION
Infinite impulse response (IIR) filters and finite impulse response (FIR) filters are two types of digital filters. In many applications it is often advantageous to employ FIR filters, since they can be designed with exact linear phase and they exhibit no stability problems. However, FIR filters are computationally more complex than IIR filters with equivalent magnitude responses, [1].
During the past several years, many design methods have been proposed to reduce the complexity of the FIR filters, specifically by reducing the number of multipliers, or using multiplier-free design, where the coefficients are reduced to simple integers or to simple combinations of powers of two, [2-4]. In this paper we propose simple technique based on the well known IFIR (Interpolated FIR) structure [2]. The coefficients of the model filter are rounded to nearest integers, which can be implemented with the adders and shifts. The images introduced by expansion of the model filter are eliminated by comb filter of the length equal to the model filter expansion factor. Additionally, another comb filter is applied in order to improve the image attenuations, where the first zero of that comb is approximately in the desired stopband frequency. The rest of the paper is organized in the following way. Section 2 briefly introduces IFIR structure and comb filters. The compensators are shown in Section 3. The proposed design is given in Section 4 and illustrated with one example.

2. REVIEW OF IFIR STRUCTURE AND COMB FILTERS
The basic idea of an IFIR structure is to design a FIR filter as a cascade of two less order filter sections. One section is the expanded model filter $G(z^M)$ and another one is the interpolator $I(z)$. The filter $G(z^M)$, where M is the interpolation factor, is obtained by introducing M-1 zeros between each

sample of the unit sample response of $G(z)$. The function of the interpolator filter $I(z)$ is to eliminate images introduced by $G(z^M)$. The IFIR structure is shown in Fig.1.

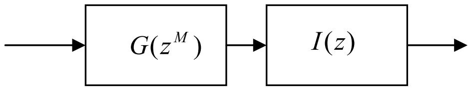


Figure 1. IFIR structure.

For a given passpand and stopband frequencies, ω_p and ω_s, of the prototype filter, the specification of the model filter is given in the following equation,

$$\omega_{G_p} = M\omega_p; \omega_{G_s} = M\omega_s. \qquad (1)$$

In that way the complexity of the model filter is approximatelly M times less than that of the prototype filter.

We proposed here to use the comb filters as the interpolators.The transfer function and the magnitude characteristic of the comb filter of the length M are given as,

$$H_{comb}(z) = \frac{1}{M}\sum_{i=1}^{M-1} z^{-i} = \frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}. \qquad (2)$$

$$\left|H_{comb}(e^{j\omega})\right| = \left|\frac{\sin(M\omega/2)}{M\sin(\omega/2)}\right|. \qquad (3)$$

As an example Fig.2 shows the magnitude characteristic of the comb filter for M=10.

The comb filter has some favorable characteristics like:

- Comb is multiplierless filter.
- It has zeros at multiple of $2\pi/M$.

Therefore the comb filter of the length M can be used to attenuate the images of the expanded model filter.To further attenuate the images we propose to use another comb filter of the length M_1 , where M_1 is defined by the stopband frequency of the prototype filter,

$$M_1 = \text{round}(2\pi/\omega_p), \qquad (4)$$

where round(x) means the rounding of x to the nearest integer.

3. COMPENSATORS

The magnitude characteristic of the comb filter exhibits a high passband droop which should be compensated. We adopt the wideband multiplier-less compensator from [5] given by the following transfer function,

$$G(z^M) = S[Bz^{-M} + Az^{-2M} + Bz^{-3M}], \qquad (5)$$

where S is the scaling factor and A and B are integer coefficients.

Table I shows the values for S, A and B for different values of K, where K is the number of the cascaded comb filters. More details can be found in [5].

4. PROPOSED METHOD

The proposed design procedure is described in the following steps:

Step 1*:* The model filter is designed using the specification from (1). Next, the impulse response $g_m(n)$ is rounded as

$$g_{mr}(n) = r\ \text{round}(g_m(n)/r), \qquad (6)$$

where r is the rounding constant, $r=2^{-6}$.

Step 2: The rounded impulse response $g_{mr}(n)$ of the model filter is expanded by M.

Step 3: The comb filters of lengths M and M_1 are designed where M_1 is defined in (4).
Step 4: Comb filters are cascaded with the expanded rounded model filter.
Step 5: The numbers of the cascaded comb filters K and K_1 are increased until the stopband specification is satisfied.
Step 6: For given K and K_1, the compensators are designed using Table 1.

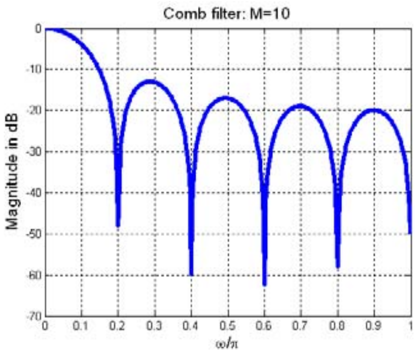


Figure 2. Comb filter.

Table 1. The values of parameters.

K	S	B	A
1	2^{-4}	-1	2^4+2^1
2	2^{-3}	-1	2^3+2^1
3	2^{-4}	$-2-2^0$	$2^4+2^2+2^1$
4	2^{-4}	-1	2^2+2^1
5	2^{-4}	-2^2-2^0	$2^4+2^3+2^1$

The overall transfer function is:

$$H(z) = G_{mr}(z^M) H^{K_1}_{comb,1}(z) H^{K_2}_{comb,2}(z) G(z^M) G(z^{M_1}), \qquad (7)$$

where K_1 and K_2 are the numbers of the cascaded comb filters $H_{comb,1}$ and $H_{comb,2}$, respectively, and $H_{rm}(z)$ is the transfer function of the rounded model filter. The method is illustrated in the following example.

Example 1: We design the multiplier-less lowpass FIR filter with the normalized edge frequencies 0.01 and 0.1, passband ripple 0.1 dB and the minimum stopband attenuation of 60 dB.
The classical design using Remez algorithm requires 30 multipliers.
Step 1. The model filter is designed and rounded using the rounding constant $r=2^{-6}$. The transfer function of the rounded model filter is,

$$G_{mr}(z)=-1+3z^{-1}+17z^{-2}+26z^{-3}+17z^{-4}+3z^{-5}-z^{-6}. \qquad (8)$$

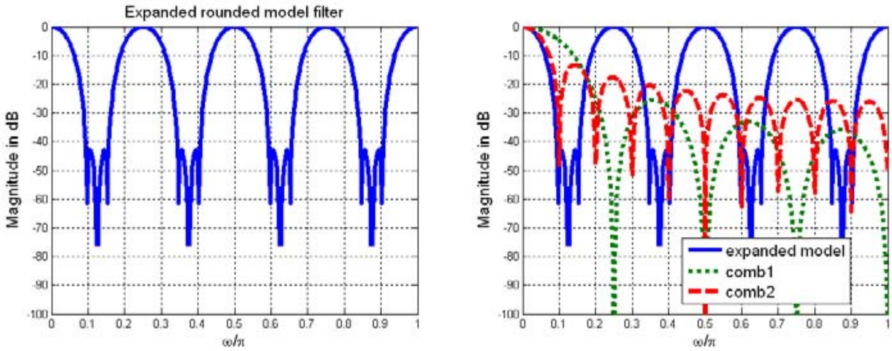
Step 2. The model filter is expanded by $M=8$.
Step 3: From (4) it follows $M_1=20$. The comb filters of lengths $M=8$ and $M_1=20$, are designed.
Step 4: The comb filters are cascaded with the expanded model filter. Figure 3a shows the expanded model filter and comb filters.
Step.5: We need $K_1=K_2=2$ to satisfy stopband specification.
Step.6: The compensator filters are designed with the parameters from the second row of Table 1. ($K_1=K_2=2$).
The transfer function is

$$H(z) = G_{mr}(z^8)\left[\frac{1}{8}\frac{1-z^{-8}}{1-z^{-1}}\right]^2\left[\frac{1}{20}\frac{1-z^{-20}}{1-z^{-1}}\right]^2 G(z^8)G(z^{20}), \qquad (9)$$

where

$$G(z^8) = 2^{-3}[-z^{-8}+(2^3+2)z^{-16}+-z^{-24}],\ \ G(z^{20}) = 2^{-3}[-z^{-20}+(2^3+2)z^{-40}+-z^{-60}]. \qquad (10)$$

The magnitude characteristic of the designed filter is shown in Fig 3b along with the passband zoom, to show that the passband specification is satisfied. We can note that the stopband specification is also satisfied.



a. *b.*

Figure 3. Expanded model filter and comb filters.

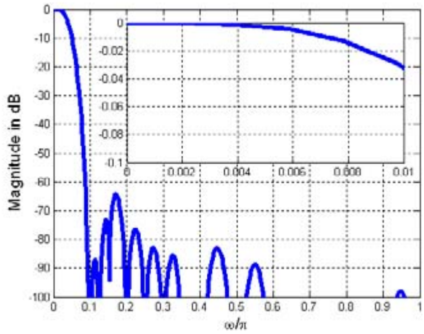


Figure 4. Overall magnitude response and passband zoom.

5. CONCLUDING REMARKS

This paper presents novel method for multiplierless lowpass FIR filter design. The method is based on IFIR structure where the coefficients of the impulse response of the model filter are rounded to the nearest integers, which can be implemented by adders and shifts. The cascaded compensated comb filters are used to eliminate images introduced by expanding the rounded model filter. The method is suitable for the narrowband filters, where $\omega_p \leq 0.1\ \omega_s$, and $\omega_s < 0.3$.The model filter is designed using the Remez algorithm. All filters are designed using MATLAB.

6. REFERENCES

[1] S. K. Mitra: Digital Signal Processing - A Computer-Based Approach, Third Edition, McGraw-Hill Inc., New York, 2006.

[2] Y. Neuvo, D. Cheng-Yu and S. K. Mitra: Interpolated Finite Impulse Response Filters, IEEE Transactions on Ac. Speech and Signal Processing, vol.ASSP-32, No.3, 1984, pp. 563–570.

[3] S. Dong, and Y. Yu: Design of Linear-Phase FIR filters with High Probability of Achieving Minimum Number of Adders, IEEE Trans on Circ and Syst. I: Regular Papers, vol.58, No.1, 2011, pp.126-136.

[4] M. Laddomada, D. Troncoso, G. J. Dolecek: Design of Multiplierless Decimation Filters using an Extended Search of Cyclotomic Polynomials, IEEE Trans. On Circ. and Systems II: Express Briefs, vol.58, No.2, 2011, pp.115-119.

[5] G. J. Dolecek and L. Dolecek: Novel Multiplierless Wide-Band CIC Compensator, International Conference IEEE ISCAS 2010, Paris, May 2010, pp. 2119-2122.