15th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2011, Prague, Czech Republic, 12-18 September 2011

MATHEMATICAL AND COMPUTER MODEL OF THE CAR DRIVER IN A BUMPY ROAD

Fehmi Krasniqi; Ramë Likaj; Ahmet Shala and Valdrin Krasniqi
Mechanical Engineering Faculty
University of Prishtina
Prishtina
Kosovo

ABSTRACT
This paper presents the system of equations with eight differential equations of the first non-linear homogenous order. MathCAD software was used to present the differential equations. However, Matlab was used to solve the differential equations and then at the end Simulink was used to build the diagram for calculating the reactions that the driver feels while driving a car in a bumpy road.
Keywords: Simulation, driver, car, linear, non-linear

1. INTRODUCTION
We have analyzed a model with four degrees of freedom. This model corresponds to a car where we analyzed the driver's vertebral reactions while driving in a bumpy road, expressed as such:

$$x_0 = 0.2 \cdot \sin(t) \ [m].$$

Figure 1. Driving the car in a bumpy road

Data:
c_g=82000 [N/m]; $c_{s,amI}$=45000 [N/m]; $c_{s,amII}$=35000 [N/m]; b_{am}=4700 [Ns/m]; M=1880 [kg]; $m_{b,I}$=55 [kg]; $m_{b,II}$=45 [kg]; m_{nj}=90 [kg]; J_C=3550 [kg/m^2]; L_1=0,9 [m]; L_2=1,1 [m]; L=0,1 [m].

By finding the coordinates of the movement, velocity, and acceleration of the respective links, we were able to get “compression” and “tension” of the vertebral column of the driver, while driving on a linear sinusoidal shaped road using the linear and non-linear model. We get the linear model by substituting $\sin x_1 \approx x_1$ for the approximate potential energy [1].

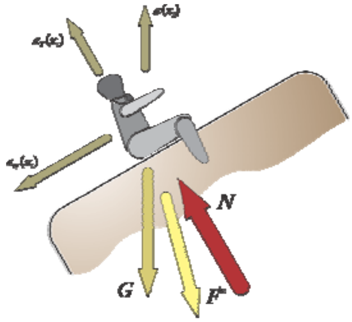

Figure 2. Mechanical Model of the normal reaction

2. FORMATION OF THE DIFFERENTIAL EQUATIONS DESCRIBING THE MOVEMENT OF THE MECHANICAL MODEL

We simplify the problem schematically as shown on Figure 3.

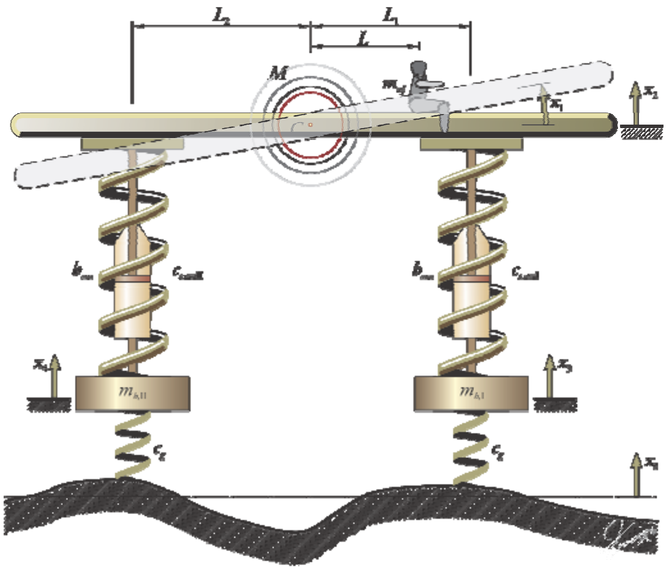

Figure 3. Car Mechanical Model

Lagrange Equation of the second type for system movements looks like this:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \mathring{x}_j}\right)+\frac{\partial E_p}{\partial x_j}+\frac{\partial D}{\partial \mathring{x}_j}=Q_j;\ \text{where } j = 1, 2, 3 \text{ and } 4. \qquad \ldots(1)$$

Using MathCAD by presenting the Lagrange equations of the second type, we can get four differential equations of the second grade [2].

3. SYSTEM OF DIFFERENTIAL EQUATIONS

After substitutions and after lowering the order of the system of equations we get eight differential equations of the first order, as such:

$$\overset{\circ}{x}_1 = x_5; \qquad \overset{\circ}{x}_2 = x_6; \qquad \overset{\circ}{x}_3 = x_7; \qquad \overset{\circ}{x}_4 = x_8.$$

$$\begin{aligned}\overset{\circ}{x}_5 \cdot \left(J_C + m_{nj} L^2\right) + c_{s,amI} \cdot \left[x_2 + L_1 \sin(x_1) - x_3\right] \cdot L_1 \cos(x_1) - \\ - c_{s,amII} \cdot \left[x_2 - L_2 \sin(x_1) - x_4\right] \cdot L_2 \cos(x_1) + \\ + b_{am} \cdot \left(x_6 + L_1 \cdot x_5 - x_7\right) \cdot L_1 - b_{am} \cdot \left(x_6 - L_2 \cdot x_5 - x_8\right) \cdot L_2 = 0;\end{aligned} \quad \ldots(2)$$

$$\begin{aligned}\overset{\circ}{x}_6 \cdot \left(M + m_{nj}\right) + c_{s,amI} \cdot \left[x_2 + L_1 \sin(x_1) - x_3\right] + c_{s,amII} \cdot \left[x_2 - L_2 \sin(x_1) - x_4\right] + \\ + b_{am} \cdot \left[2x_6 - x_5 (L_1 + L_2) - x_7 - x_8\right] = 0;\end{aligned} \quad \ldots(3)$$

$$\begin{aligned}\overset{\circ}{x}_7 \cdot m_{b,I} - c_{s,amI} \cdot \left[x_2 + L_1 \cdot \sin(x_1) - x_3\right] + c_g \cdot \left[x_3 - x_0(t)\right] - b_{am} \cdot \left(x_6 + L_1 \cdot x_5 - x_7\right) = 0; \\ \overset{\circ}{x}_8 \cdot m_{b,II} - c_{s,amII} \cdot \left[x_2 - L_2 \cdot \sin(x_1) - x_4\right] + c_g \cdot \left[x_4 - x_0(t)\right] - b_{am} \left(x_6 - L_2 \cdot x_5 - x_8\right) = 0.\end{aligned} \quad \ldots(4)$$

4. SOLVING THE SYSTEM OF DIFFERENTIAL EQUATIONS

We can find the differences between the linear and the non-linear model using the diagram, as shown below:

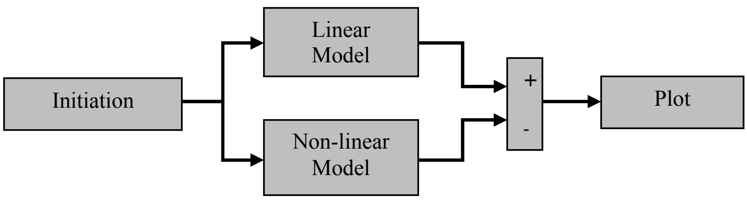

Figure 4. The diagram for finding the difference between the linear and the non-linear model

Now that we know the movements of the system, respectively the velocity of the system [3], we can build the simulation diagram for calculating the difference between the static and the "dynamic" mass that the driver feels during the movement, as shown below:

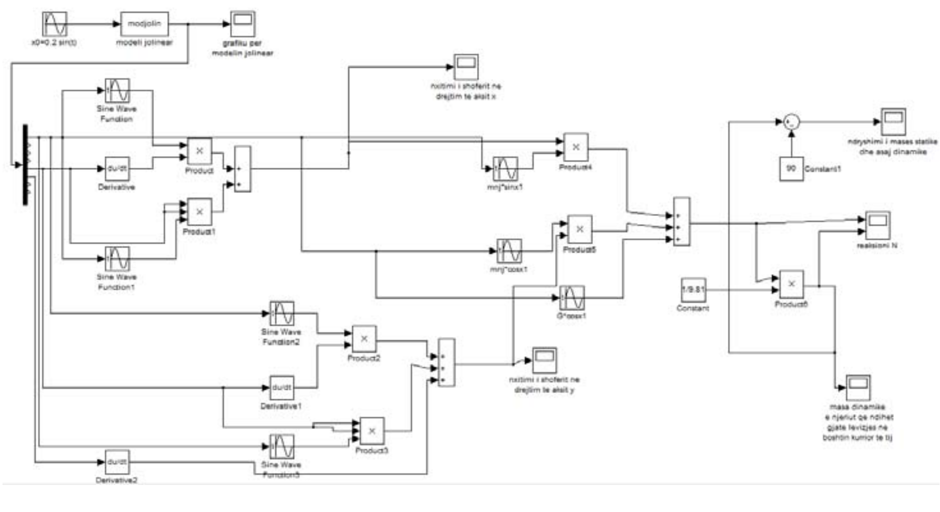

Figure 5. Simulation diagram

5. RESULTS

After the execution, we get the normal reaction as the one in Figure 6. Also the driver's vertebral movements while driving in a bumpy road are also shown in the Figure 7.

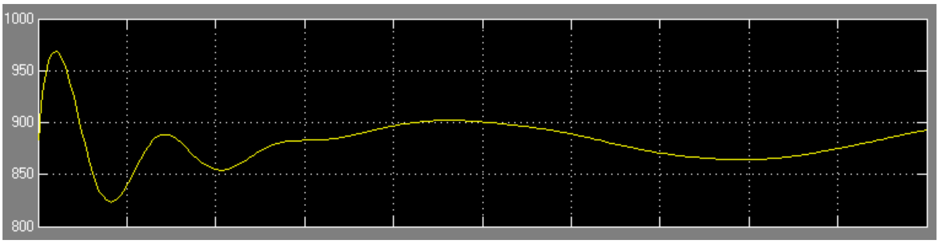

Figure 6. Reaction N in the normal direction of the car frame

The Figure 7 below shows how the driver feels this bumpy road. Over the long run this is not healthy for the driver because it acts directly on the vertebral column through tension and compression of the actual vertebra. The value of this tension and compression is close to 4 kilograms difference.

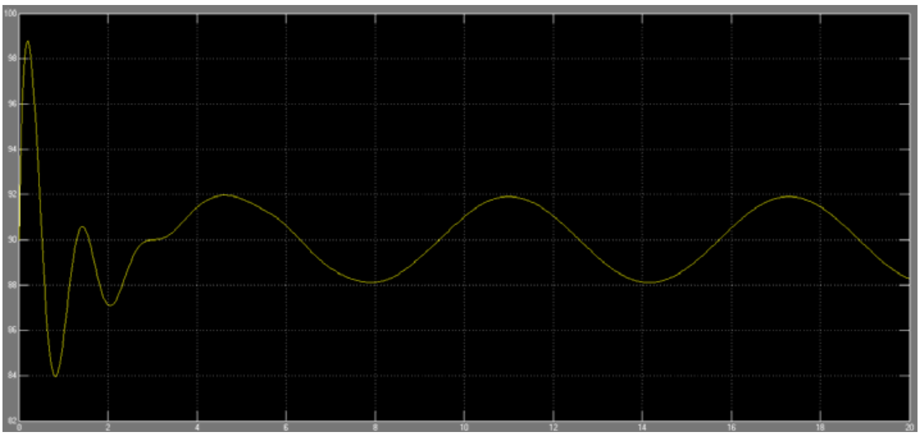

Figure 7. The differences felt in the driver's vertebral column while driving in a bumpy road

6. CONCLUSION AND RECOMMENDATIONS

In this project, we presented an example of the driver of the car in a bumpy road of the sinusoidal form. We wanted to present the differences that the driver feels while driving in this road. The differences are very apparent in the diagrams above, as it is impossible to cushion these movements completely. We presented the difference in the weight of the driver, or simply the difference in normal reaction in the driver's seat as e result of the masses and inertia forces during the car movement.

7. REFERENCES

[1] Dujaka, Aziz, Biomekanika, Prishtina, 2010
[2] Lika, Ramë, Simulimi i sistemeve mekanike, Prishtina, 2007
[3] Shala, Ahmet (2009): Softwaret Aplikativ, Prishtinë