CONTENTS AUTHORS

15th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2011, Prague, Czech Republic, 12-18 September 2011

EFFECT OF WEB ARRANGEMENT ON THIN-RIM GEAR TOOTH CONTACT STRESS

Milan Opalić, Krešimir Vučković
Faculty of Mechanical Engineering and Naval Architecture
Ivana Lučića 5, 10000 Zagreb
Croatia

Milan Kljajin
Mechanical Engineering Faculty in Slavonski Brod
Trg Ivane Brlić-Mažuranić 2, 35000 Slavonski Brod
Croatia

ABSTRACT
Thin-rim gears are often used in applications where light weight and compact design are demanded. Since the gear rim is more rigid in the area of the gear web, this influences the load as well as the contact stress distribution along the tooth face width. Conventional standard procedures for gear design assume uniform rigidity along the face width and therefore are not fully applicable to the thin-rim gears tooth contact stress calculation. The objective of this paper is to evaluate effect of web arrangement on a thin-rim gear tooth contact stresses using advanced engineering tools for a numeric analysis. Three-dimensional parametric finite element model of thin-rim gear engaged with a solid gear is created. The gear pair contact position is chosen to load the thin-rim gear (the wheel) at the highest point of single tooth contact assuming small sliding and dry frictionless contact. Commercial finite element package Abaqus/Standard is used to determine contact stresses assuming small displacement hypothesis. The material is considered homogenous and isotropic with a linear elastic behavior. Obtained tooth contact stress contour lines as well as the maximum tooth contact stress values for various thin-rim gear web arrangements are presented.
Keywords: thin-rim gear, tooth contact stress, finite element analysis, web arrangement

1. INTRODUCTION
A design goal for the gears in aircraft power transmissions is to transmit high loads at high speeds with both size and weight kept to a minimum. To achieve this goal, some gear designs incorporate thin rims linked with the hub by a solid thin web. Since the gear rim is more rigid in the area of the gear web, this influences the load as well as the contact stress distribution along the tooth face width. The most common tooth contact pressure calculation methods are based on conventional standard procedures for gear design, published by German Institute for Standardization (DIN), American Gear Manufacturers Association (AGMA) and International Organization for Standardization (ISO). These procedures are derived from two-dimensional cantilever-beam model assuming uniform rigidity along the tooth face width and therefore are not fully applicable to the thin-rim gears tooth contact stress calculation.
A number of researchers have investigated the effects of various design parameters on thin-rim gears stresses using finite element (FE) analysis; however they were primarily focused on tooth root. Notable differences for the tooth root maximum principal stress versus backup ratio (rim thickness divided by tooth height) curves are shown in comparison study between various references [1]. These differences resulted from differences in tooth geometry, mesh refinement and more notable differences in support conditions. Effect of web arrangement on tooth root stresses was investigated in [2]

concluding that three-dimensional analysis is required. Tooth contact [3] and tooth root stress analyses [4], both on the basis of whole gear model, showed that partial deformation model is not suitable for thin-rim gears with asymmetrical web arrangement if the rim is fixed on the boundary. Similar conclusion was obtained in study [5] for thin-rim gears with symmetrical web arrangement. Studies [3, 4] also revealed that tooth contact stress distribution is not uniform along the thin-rim tooth face width. Effect of rotational speed on the tooth contact stresses of thin-rim gear with asymmetrical web arrangement was investigated in studies [6, 7]. It has been established that the increase in the gear's rotational speed, due to the effect of rotational body force (centrifugal load), changes tooth contact pattern and decreases the maximum tooth contact stress value. This happens because rotational body force opens the thin-rim gear, by pushing the free end of the rim toward the mating gear and thus providing the additional support for the mating gear tooth [7].
The objective of this paper is to evaluate effect of web arrangement on a thin-rim gear tooth contact stresses using a nonlinear FE analysis. Obtained tooth contact stress contour lines as well as the maximum tooth contact stress values for various thin-rim gear web arrangements are presented.

2. NUMERICAL MODEL

Gear pair used in this study is modeled assuming ideal conditions; i.e., no tolerances, clearances, assembly and machining errors. The pinion is a solid gear and the wheel is a thin-rim gear. Their geometrical and material parameters are given in Table 1.

Table 1. Parameters of gears

Parameter	Symbol	Pinion	Wheel
Number of teeth	z	18	50
Module	m	4 mm	4 mm
Normal pressure angle of basic rack	α	20°	20°
Profile shift coefficient	x	0.3140	0.1688
Root fillet radius of basic rack	ρ_{fP}	$0.2 \cdot m$	$0.38 \cdot m$
Addendum of basic rack	ρ_{aP}	$1 \cdot m$	$1 \cdot m$
Dedendum of basic rack	ρ_{fP}	$1.25 \cdot m$	$1.25 \cdot m$
Tip diameter	d_a	82.332 mm	209.171 mm
Tooth face width	b	40 mm	40 mm
Web inner diameter	-	-	85 mm
Rim thickness	s_R	-	5 mm
Web thickness	b_s	-	4 mm
Web offset from the middle of tooth face width	-	-	0, 10 and 18 mm
Young's modulus	E	210 000 MPa	210 000 MPa
Poisson's ratio	ν	0.3	0.3

The commercially available FE software *Abaqus 6.7.3* is used for analysis [8]. Pre and post processing are done in *Abaqus/CAE* and numerical analysis in *Abaqus/Standard*. A thin-rim gear is modeled as the whole gear deformation model while the mating gear is modeled as a partial deformation model having six teeth. The gear pair contact position is chosen to load the thin-rim gear at the highest point of single tooth contact (HPSTC). The material is considered homogenous and isotropic with a linear elastic behavior; material properties are given in Table 1. Small displacement hypothesis is assumed for the purpose of the analysis. The gear models are partitioned so that a structured meshing technique could be applied. The mesh is refined around the mating teethes, and especially near the geometrical contact line at HPSTC, as shown in Figure 1a. Incompatible modes hexahedral linear elements are used for the mesh. The contact between mating teeth flanks is considered a dry frictionless contact, assuming small sliding between them. The contact constraint is enforced with a Lagrange multiplier method using *Abaqus* option *hard contact*. To impose kinematic and static boundary condition, the mating gear (pinion) inner cylindrical surface and two radial planar surfaces are fixed. Kinematic coupling constrain is used in order to prescribe a torque to a gear's web inner cylindrical surface without constraining the circular displacement, but constraining the radial and the axial ones. The partitioned three-dimensional model of engaged gears along with applied torque and boundary conditions is shown in Figure 1b.

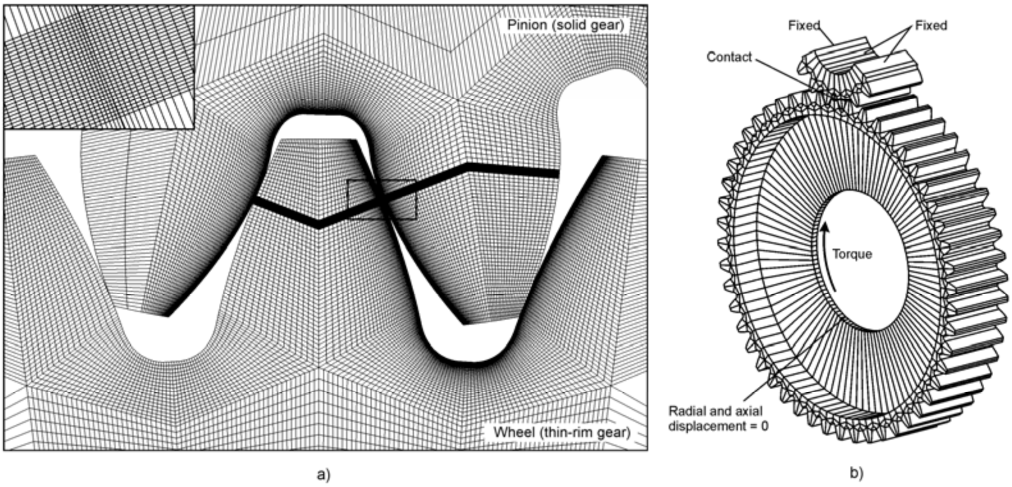

Figure 1. FE model: a) refined mesh detail and b) applied torque and boundary conditions

In total, four cases are considered. The first one included the pinion engaged with a solid gear wheel in order to validate the proposed numerical model by comparing obtained maximum contact stress value with the one calculated according to Hertz. The remaining three cases included thin-rim gear as a wheel, with web positioned: in the middle, at the end and between the middle and the end of the face gear width. The corresponding web offsets from the middle of tooth face width along with web thickness are given in Table 1. The wheel is loaded with torque value of 300 Nm in all cases.

3. RESULTS

Uniform contact stress distribution along the face width is obtained for the solid gear wheel with the small deviation near the sides of the tooth due to the free material expansion, as shown in Figure 2a. The obtained maximum tooth contact stress for prescribed torque value of 300 Nm is 574 MPa which correlates very well to the value of 570 MPa calculated according to Hertz.

It is found that contact stress distributions for thin-rim gears are not uniform along the face width, as shown in Figures 2b, 3a and 3b. Compared to the solid gear, the tooth contact stress distribution for the thin-rim gear with the web positioned in the middle of tooth face width is slightly changed from the uniform into a center contact, with relatively small increase of maximum tooth contact stress value from 574 to 593 MPa, see Figure 2. Compared to the solid gear, the tooth contact stress distributions for the thin-rim gears with offset web significantly changed from the uniform into a side heavy contact, as shown in Figure 3.

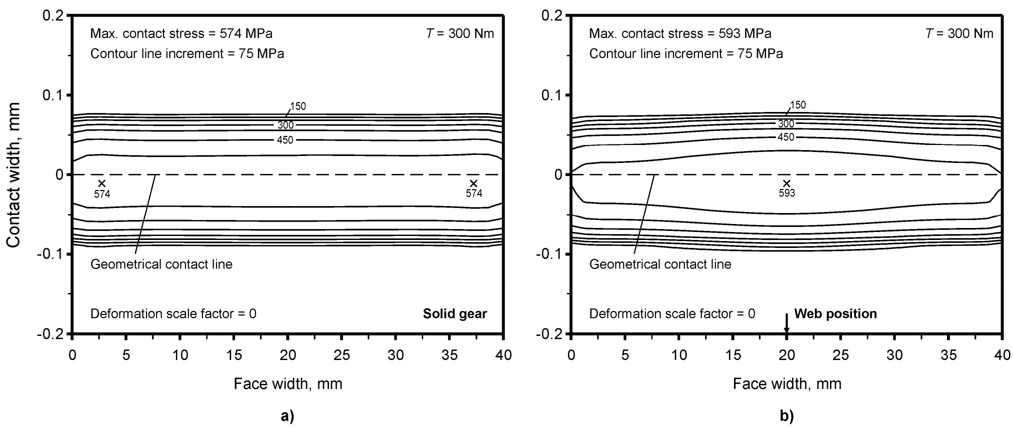

Figure 2. Tooth contact stress contour lines for: a) solid gear and b) thin-rim gear with middle web

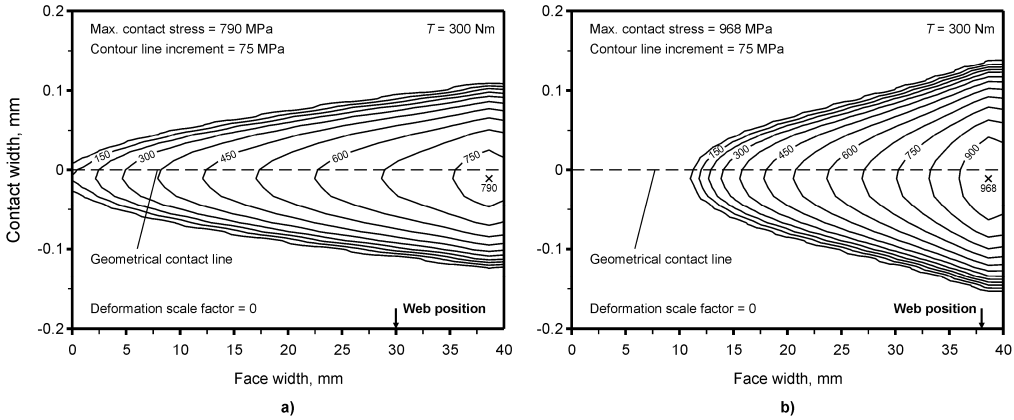

Figure 3. Tooth contact stress contour lines for thin-rim gear with web positioned: a) between the middle and the end and b) at the end of tooth face width

In the case of web positioned between the middle and the end of the tooth face width, the maximum tooth contact stress value is increased from 574 MPa to 790 MPa, see Figure 3a. In the case of the web positioned at the end of the tooth face width it is increased even more to a value of 968 MPa, see Figure 3b. In all cases considered, location of the maximum contact stress was shifted toward the gear tooth root regarding the geometrical contact line, presumable due to the deformations of the teeth.

4. CONCLUSION

Effect of web arrangement on a thin-rim gear tooth contact stresses is evaluated in the framework of the nonlinear FE analysis. The proposed numerical model is validated by comparing obtained maximum contact stress value for two solid gears in contact at HPSTC with the one calculated according to Hertz. The obtained results for analyzed thin-rim gears show that web arrangement has influence on maximum tooth contact stress including its value and position causing non-uniform contact along tooth face width. The smallest deviation from uniform contact is found for thin-rim gear with web positioned in the middle of the tooth face width with maximum tooth contact stress increase of negligibly 3.3 %. The largest deviation is found for thin-rim gear with web positioned at the end of the tooth face width with maximum tooth contact stress increase of notably 68.6 %. We can conclude that in cases where lightweight gear with offset web is required the web should be positioned as closely as possible to the middle of the gear face width. In regard to tooth contact stresses, the proposed numerical model is an effective tool that can evaluate whether gear design is satisfactory or not. Obtained results can also serve as basis for the tooth face micro-geometry modifications.

5. REFERENCES

[1] Bibel, G.D., Reddy, S.K., Savage, M.: Effects of Rim Thickness on Spur Gear Bending Stress, NASA TM-104388, AVSCOM TR-91-C-015, 1991.,

[2] Opalić, M., Kolaček, Z.: Tooth Root Stress Analysis of Thin Rim Gear, DMMI '99, Journal of Mechanical Engineering (special edition), 1999.,

[3] Li, S.: Gear Contact Model and Loaded Tooth Contact Analysis of a Three-Dimensional, Thin-Rimmed Gear, Journal of Mechanical Design, Vol. 124., 2002.,

[4] Li, S.: Deformation and Bending Stress Analysis of a Three-dimensional Thin-rimmed Gear, Journal of Mechanical Design, Vol. 124., 2002.,

[5] Conrado, E., Davoli, P.: The True Bending Stress in Spur Gears, Geartechnology, Vol. 24, No. 6, 2007.,

[6] Li, S.: Centrifugal load and its effects on bending strength and contact strength of a high speed thin-walled spur gear with offset web, Mechanism and Machine Theory, Volume 43, Issue 2, pp 217-239, 2008.

[7] Vučković, K, Ulbin, M. Belšak, A.: FEM Model for The Analysis of Rotational Speed Influence on Tooth Contact Pressure Distribution of Thin-Rimmed Gears with Asymmetric Web Arrangement, DESIGN 2008, Proceedings of DESIGN 2008 - 10th International Design Conference, Dubrovnik, Croatia, 2008.,

[8] Hibbitt, Karlsson & Serensen, Inc.: Abaqus/Standard. User's guide & theoretical manual, *Ver. 6.7.3,* 2007.