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MATHEMATICAL MODEL OF TOOL DIAMETER CALCULATION

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ABSTRACT
This paper describes the mathematical model used to determine the front diameter of the tool for pressing grooves by cold rifling, depending on: tool geometry, the degree of deformation, type and condition of material.
Keywords: Tool Geometry, Multistage Tools, Front Diameter, Deformation Ratio, Hardening Curve

1. INTRODUCTION
According to stress analysis of rifling, one can conclude that the injection process is significantly affected by tool geometry. The results of previous research, and experimental verification, confirm this fact. In order to define the geometry of the tools which provide the optimal technology, we have described some well-known methods for calculation of tool diameter. Analysis for calculating tool diameter, will be performed under the assumption that rifling is done by conical tool without grooves.

2. DIAMETER OF THE TOOL FOR RIFLING USING MULTISTAGE TOOL
Diameter of the front ring for rifling with wall thickness reduction, which can analogously be applied to the two-stage tool for rifling can be described by the following equation:

$$d_{a1} = \sqrt{\left(d_{an}{}^{2} - d^{2}\right)\cdot(1-\Psi)\frac{1}{2} + d^{2}}\ . \qquad \text{... (1)}$$

where: d_{a1} - diameter of the front portion of the tool,
d - internal diameter of the sample,
d_{an} - tool diameter for rifling stage "n",
ψ - contraction of the cross section of the work piece after pulling the front end of tool,
A_o - starting area of work piece cross section
A_1 - area of work piece cross section after pulling front end of tool.

In order to be able to calculate the diameter of the front end of the tool according to equation (1), we should adopt a value of contraction of the cross section, thus affecting the outcome beforehand.

2.1. Setting up the model for calculating tool diameter, rifling by multistage tools
In order to solve the problem of determining diameter of the front end of the multistage rifling tools, we start from the conditions of equal degree of wear [5]. The requirement of equal degree of reduced wear is based on the condition of equality of deformation work per each tool stage. In the general case, the total deformation work of indentation grooves in the rifling tool is equal to the product of force and rifling path. Using the expression for the rifling force, we get the equation for deformation work in the form of:

$$W = V \cdot k\varphi_z\left[\left(\frac{1}{2}+\frac{\mu}{\alpha}\right)+\frac{2}{3}\frac{\alpha}{\varphi_z}\right]. \qquad \text{... (2)}$$

where: V - work piece volume, α - tool cone angle; μ - friction factor;

By dividing the equation (2) by volume, we obtain specific deformation work:

$$a = k\varphi_z\left[(\frac{1}{2}+\frac{\mu}{\alpha})+\frac{2}{3}\frac{\alpha}{\varphi_z}\right]. \qquad ... (3)$$

The expression (3) can be broken down into three parts, so that the specific strain work consists of:
- specific work of deformation, which is spent on changing the shape [1]:

$$a_1 = \frac{k\varphi_z}{2} = k_s\varphi_z . \qquad ... (4)$$

- specific work of deformation, which is spent on bending the fibers from rifling direction, the direction under the angle and returning in the direction of rifling on the exit of the deformation zone:

$$a_2 = \frac{2}{3}k\alpha . \qquad ... (5)$$

- specific work of deformation of friction

$$a_3 = k\cdot\varphi_z\cdot\frac{\mu}{\alpha}. \qquad ... (6)$$

By applying the condition of equality of the specific deformation work at all stages of a rifling tool, which are spent to change the shape, we can write:

$$a_{1i} = k_i\cdot\varphi_{zi} = const . \qquad ... (7)$$

In order to obtain an equal degree of wear for all stages of rifling tool, one must provide equal temperature rise at each level.
Due to the conversion of mechanical to thermal energy at higher strain rate assuming adiabatic process, the temperature rises as follows:

$$t_{1i} = \frac{a_{1i}}{c\cdot\gamma} = \frac{k_i\cdot\varphi_{zi}}{c\cdot\gamma} = const . \qquad ... (8)$$

where: c - specific heat of the work piece; γ - work piece density; i - number of tool stages
The expression (8) is reduced to precondition of equality of specific deformation work (7), which in general case of rifling with multistage tool with n stages reads:

$$k_1\cdot\varphi_{z1} = k_1\varphi_{z2} = k_3\varphi_{z3} = ... = k_n\cdot\varphi_{zn} . \qquad ... (9)$$

where: k_1 - k_n - specific strain resistance after rifling, n - the stage of tool
$\varphi_{z1}\div\varphi_{zn}$ - logarithmic deformation rate for the first, i.e. n^{th} stage.
In a variety of papers [2], [3],[4]and [5] it is common to use the following expression, derived from multiple experiments, as analytical approximation of hardening curve of a third degree:

$$k = C\cdot\varphi_z^{\ n} . \qquad ... (10)$$

where: C *and* n - constants whose values depend on the type of material.
Expression (10) provides a link between the specific deformation resistance and the logarithmic strain, and can be used to eliminate specific deformation resistance k from the equation (9).
Introducing the expression (10) into (9), we obtain:

$$\begin{aligned}&\varphi_{z1}^n\cdot\varphi_{z1} = (\varphi_{z1}+\varphi_{z2})^n\cdot\varphi_{z2} = \\ &= (\varphi_{z1}+\varphi_{z2}+\varphi_{z3})^n\cdot\varphi_{z3} = ... \\ &= (\varphi_{z1}+\varphi_{z2}+...+\varphi_{zn})^n\cdot\varphi_{zn}{}^n\end{aligned} \qquad ... (11)$$

Known values in (11) include hardening exponent (n) and the total logarithmic strain ratio (φ_{zu}):

$$\varphi_{zu} = \varphi_{z1}+\varphi_{z2}+\varphi_{z3}+...+\varphi_{zn} . \qquad ... (12)$$

and we should determine the value of the logarithmic degree of deformation in the front stages of tool $(\varphi_{z1},\varphi_{z2},\varphi_{z3,}........\varphi_{zn-1})$.
The value of logarithmic strain rate, or redistribution of the total strain rate for multistage tools are given in Table 1 and in the first diagram. Based on the solutions of the system of equations [5], it is possible to determine the diameter of the front stage tools for rifling using the form:

$$d_{ai} = \sqrt{\frac{4 \cdot A_i}{\pi}} \qquad \text{... (13)}$$

where: A_i - cross-sectional area of the work piece mouth after rifling the i[th] tool stage. The area A_i in equation (13) should be expressed through the known values, using the solutions of equations [5]:

$$A_i = A_0^{C_1} \cdot A_n^{C_2} \qquad i = 1 \div (n-1) \qquad \text{... (14)}$$

where: n - number of tool stages;

$A_o = d^2\pi/4$ - initial cross-sectional area of the work piece mouth where the grooves are rifled;

$A_n = d_a^2\,\pi/4$ - final cross-sectional area of the work piece mouth where the grooves are rifled.

Substituting expressions (14) in (13) gives a general equation for calculating the frontal rifling tool diameter:

$$d_{ai} = \sqrt{\frac{4}{\pi} A_o^{C_i} \cdot A_n^{C_2}} \qquad \text{... (15)}$$

Constants C_1 and C_2 as a function of logarithmic strain rate, are given in Table 2 and Figure 2.

For two-stage tool: $C_1 = \dfrac{\varphi_2}{\varphi_u}$; $\qquad C_2 = 1 - C_1$

2.2. Influence of materials on the value of tool front stage diameter

The model for calculating the diameter of the front stage rifling tool diameter in the multistage rifling process, depends on the hardening exponent "n". The values of hardening exponent curve of the third order theory is in the range $0 < n \leq 1$. Starting expression (11) when $n = 0$, becomes:

$$\varphi_1 = \varphi_2 = \varphi_3 = \ldots\ldots = \varphi_n \qquad \text{... (16)}$$

Total logarithmic strain rate in this case is evenly distributed at all levels of multistage tools. Using expression (16), the equation for calculating the diameter of the front stage rifling tool in the case of $n = 0$, reads:

$$d_{ai} = \sqrt{d^2 + \frac{4}{\pi} \qquad A_n \cdot e^{\frac{n-i}{n}\cdot\varphi_u}} \qquad \text{... (17)}$$

where: i - ordinal number of the front stage, n - total number of stages

For exponent values $0 < n < 1$, the hardening curve has raising trend, which represents the real materials. Based on the above expressions, the following equation can be set up:

$A_i = A_n \cdot \dfrac{\varphi_i}{\varphi_u}$; where:

A_i is the area of groove in the i[th] tool stage, A_n is the area of the groove in the n[th] tool stage;

$A_i = \left(d_{ai}^2 - d^2\right)\dfrac{\pi}{4}$; $A_n = \left(d_{an}^2 - d^2\right)\dfrac{\pi}{4}$; $(d_{ai}^2 - d^2)\dfrac{\pi}{4} = (d_{an}^2 - d^2)\dfrac{\pi}{4}\dfrac{\varphi_i}{\varphi_u}$

$$d_{ai} = \sqrt{d^2 + \left(d_{an}^2 - d^2\right)\frac{\varphi_i}{\varphi_u}}; \qquad \text{... (18)}$$

2.3. Setting up the model for calculating rifling tool diameter, based on the strain rate of the wall thickness

The expression for the strain rate through the thickness of the wall reads $\varphi_u = \ln\dfrac{R-r}{R-r_a} = \ln\dfrac{D-d}{D-d_a}$

then the above expression for the diameter of the i[th] tool stage has the following form:

$$e^{\varphi} = \frac{D-d}{D-d_a} \qquad d_a = D - \frac{D-d}{e^{\varphi}} \qquad {}_{ai} = D - \frac{D-d}{e^{\left(\frac{\varphi_i}{\varphi_u}\right)\varphi_u}}; \qquad \text{... (19)}$$

Values $\dfrac{\varphi_i}{\varphi_u}$ depend on work piece material and from the number of tool stages.

These values are given in Table 1 [5]. For $n = 0$: $\dfrac{\varphi_i}{\varphi_u} = \dfrac{\varphi_1}{\varphi_u} = \dfrac{\varphi_2}{\varphi_u} = \ldots = \dfrac{1}{N}$.

Table 1. Redistribution of the total strain rate for the two-stage tool.

	hardening exponent "n" of the third order curve										
φ_i / φ_u	0	0,1	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50	1,0
φ_1 / φ_u	0,500	0,516	0,524	0,531	0,538	0,545	0,552	0,558	0,564	0,570	0,618
φ_2 / φ_u	0,500	0,484	0,476	0,469	0,462	0,455	0,448	0,442	0,436	0,430	0,382

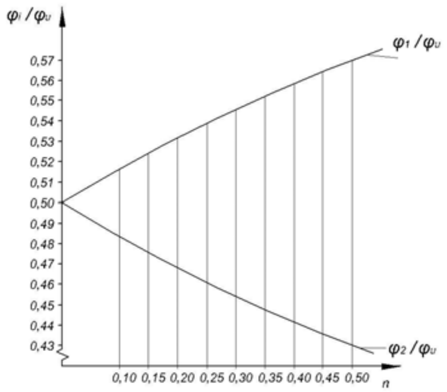

Figure 1. Total redistribution of strain rate for the two-stage tool

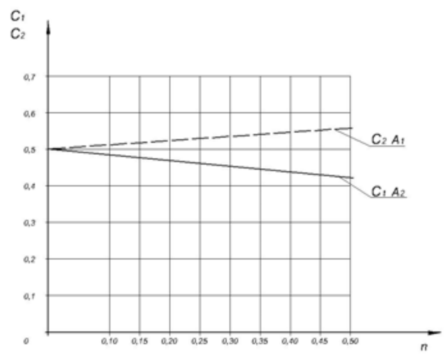

Figure 2. Hardening exponents for the two-stage tool

Figure 1 shows the redistribution of the total strain rate depending on the "n" hardening exponent of the third order curve and the from the tool stage. Figure 2 shows the dependence of the coefficient C_1 and C_2 of the hardening exponent "n" of the third order curve depending on the tool stage.

Table 2. Hardening exponents for the two-stage tool.

		hardening exponent "n" of the third order curve										
$A_i = A_0^{C_1} A_2^{C_2}$		0	0,1	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50	1,0
	C_1	0,500	0,483	0,476	0,468	0,461	0,455	0,448	0,442	0,436	0,430	0,392
A_1	C_2	0,500	0,517	0,524	0,532	0,539	0,545	0,552	0,558	0,564	0,570	0,618

Using the results given in Table 2, diameters of rifled multistage tool for all types of materials can be calculated, provided that the value of hardening exponent "n" of the third order curve is known for material from which the sample is made.

3. CONCLUSION

Based on the geometry of the tool, type and condition of material at a balanced level of tool wear, a mathematical model can be found for determining the diameter of the multistage cold rifling tool.

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