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PARAMETRIC PROGRAMMING FOR THE TRANSPORTATION PROBLEM

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ABSTRACT
The transportation problem is a linear programming model used to determine the optimal program for distribution of certain types of goods from different supply points to different demand points, while it is implied that there is a distance between the points.
In this study, a parametric model of the transportation problem will be presented, in which the coefficients of the objective function depend on a parameter. The basic question to be answered is what impact on the optimal solution to the problem has a change in values of the objective function, that is, within which interval can the coefficients of the objective function of the transportation problem vary without their affecting the calculated optimal solution.
Keywords: Parametric programming, transportation problem, objective function, optimal solution

1. INTRODUCTION

The transportation problem is a special case of linear programming models which is used to determine the optimal programme for distribution of certain types of goods, from different supply points (sources) to different demand points (destinations), while it is implied that there is a certain distance between the points. The most frequent criterion for optimization of the goods transportation programme is the total transportation costs minimization, although the criterion could be defined differently.
As a special case of the linear programming task, the objective function in the transportation problem represents the total transportation costs, while the constraints are determined by the supply and demand of certain sources, or destinations.

2. THE STANDARD FORM OF THE TRANSPORTATION PROBLEM

In order to define the standard form of the transportation problem model, we will assume that there is a finite number of m sources (supply points) - $P_1, P_2, \ldots, P_m$, with a homogeneous type of goods, for the use of which there is a strong need (demand) at n destinations (demand points) - $T_1, T_2, \ldots, T_n$.

Let us denote by:

- x_{ij} – the quantity of goods to be transported from the i-th source to the j-th destination (i = 1, ..., m; j = 1, ..., n);
- c_{ij} – transportation costs per unit of goods transported from the i-th source to the j-th destination;
- a_i – the available quantity of goods (supply) at the i-th origin (i = 1 ,..., m);
- b_j – the demand for goods observed at the j-th demand point (destination) (j = 1 ,..., n)

Methodologically speaking, the main objective of the transportation problem can be formulated as a request for determination of the optimal values of the variables x_{ij} (i=1,…,m; j=1,…,n) i.e. the optimal amount of goods transported on individual roads, for which the minimum value of the total transportation costs will be achieved, that is the minimum value of the objective function.

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad \text{...(1)}$$

while the following constraints sets must be satisfied:

a) the total amount of goods available from each source has to be distributed to the demand points, i.e.

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i=1, 2,\ldots,m \qquad \text{...(2)}$$

b) the demand of each destination has to be fully satisfied, i.e.

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j=1,\ldots,n \qquad \text{...(3)}$$

c) the quantity of goods transported via certain roads, or the appropriate variables must be non-negative values, i.e.

$$x_{ij} \geq 0 \quad i = 1,\ldots,m \quad j = 1,\ldots,n \qquad \text{...(4)}$$

3. THE PARAMETRIC TRANSPORTATION PROBLEM

In real terms, the linear programming task, and, therefore, the transportation problem, the coefficients appearing in the model are often not constant. The coefficients can be:

a) Random variables with the distribution law known. These are the tasks of the stochastic programming.

b) Variables depending on a certain parameter with the possible limits within which it may vary given. These are the tasks of parametric programming.

The basic question that arises in parametric programming is what impact a change of values of coefficients in the objective function or in the constraints set has on the optimal solution.

Introducing the parameter λ into the objective function (1) of the transportation problem, we obtain the objective function of the parametric transportation problem (5), as follows

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} (c'_{ij} + \lambda c''_{ij}) x_{ij} \qquad \text{...(5)}$$

where c is a constant vector and λ an arbitrary parameter, which determines the objective function. The problem is reduced to testing the dependence of the optimal cost solution to the transportation problem from a change in the value of the parameter λ .

The algorithm for solving the parametric transportation problem consists of several steps.

Step I: First you need to determine the initial basic solution to the transportation problem (5), for $\lambda = \lambda_0$, with constraints (2) - (4), using the potential method [2].

Step II: To test the optimality of the initial basic solution, using the potentials u'_i, u''_i, v'_j, v''_j, determined from the relation

$$\begin{aligned} c'_{ij} - u'_i - v'_j &= 0 \\ c''_{ij} - u''_i - v''_j &= 0 \end{aligned} \qquad \text{...(6)}$$

Step III: Since $c_{ij} = c'_{ij} + \lambda c''_{ij}$, multipliers γ_{ij} can be expressed as linear functions depending on the parameters, λ, i.e. $\gamma_{ij} = \alpha_{ij} + \lambda \beta_{ij}$. Hence, the condition for the optimal solution to the parametric transportation problem will be $\gamma_{ij} = \alpha_{ij} + \lambda \beta_{ij} \geq 0,\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,n$. Thus, from the conditions $(c'_{ij} + \lambda_0 c''_{ij}) - (u_i + v_j) \geq 0$, it follows that

$$\begin{aligned} \alpha_{ij} &= c'_{ij} - u'_i - v'_j \\ \beta_{ij} &= c''_{ij} - u''_i - v''_j \end{aligned} \qquad \text{...(7)}$$

Step IV: Determine the interval in which the parameter λ can vary so that the structure of the optimal solution remains unchanged. The interval for the parameter λ is determined as follows:

-If all coefficients fulfil the condition that $\beta_{ij} < 0$ the interval is obtained from the relation $-\infty < \lambda \le \min\left(-\frac{\alpha_{ij}}{\beta_{ij}}\right)$ ... (8)

-If all coefficients fulfil the condition that $\beta_{ij} > 0$ the interval is obtained from the relation $\max\left(-\frac{\alpha_{ij}}{\beta_{ij}}\right) \le \lambda < +\infty$ ... (9)

-If some coefficients fulfil the condition that $\beta_{ij} > 0$ and some that $\beta_{ij} < 0$, the corresponding interval of the parameter values is

$$\max_{\beta_{ij}>0}\left(-\frac{\alpha_{ij}}{\beta_{ij}}\right) \le \lambda \le \min_{\beta_{ij}<0}\left(-\frac{\alpha_{ij}}{\beta_{ij}}\right) \quad ...(10)$$

Step V: It is necessary to obtain a new distribution of quantities transported, i.e. to determine an improved solution of the parametric transportation problem.

The optimal solution is obtained if the condition that $\gamma_{ij}(\lambda_0) = (c'_{ij} + \lambda_0 c''_{ij}) - (u_i + v_j) \ge 0$ is satisfied, for all the free fields, i.e. for the fields via which the goods are not transported.

4. A NUMERICAL EXAMPLE

From three ponds, located in the vicinity of Podgorica, and with the daily capacities of 100, 120 and 80 kg, three fish stores are supplied with the daily demands of 60, 120 and 120 kg of fish respectively. The transportation cost matrix is

$$C = \begin{bmatrix} 10 & 12-2\lambda & 20 \\ 4+8\lambda & 2 & 4+4\lambda \\ 8-6\lambda & 8 & 10+6\lambda \end{bmatrix}$$

It is necessary to determine the optimal solutions to the given parametric transportation problem, for all the parameter values within the interval $0 \le \lambda \le 3$.

Step 1: For $\lambda = 0$, using the potential method, the initial basic solution shown in Table 1 is obtained

Table 1.

	P1	P2	P3	Capacity
S1	10 **60** 0	12 **40** -2	20 0	100
S2	4 8	2 **80** 0	4 **40** 4	120
S3	8 -6	4 0	10 **80** 6	80
Demand	60	120	120	300

Steps 2 and 3: The potential values for Table 1, should be calculated from the condition (6), assuming that the potential values are $u'_1 = 0$ and $u''_1 = 0$ and the values for α_{ij} and β_{ij} should be calculated from the condition (7). Thus, it is obtained that

$$u'_1 = 0;\ u'_2 = -10; u'_3 = -4;\ v'_1 = 10;\ v'_2 = 12; v'_3 = 14;$$
$$u''_1 = 0;\ u''_2 = 2; u''_3 = 4;\ \ v''_1 = 0; v''_2 = -2; v''_3 = 2.$$

Step 4: Since among the values β_{ij} there are both positive and negative ones, the limits of the parameter interval are obtained from the relation $\max_{\beta_{ij}>0}\left(-\frac{\alpha_{ij}}{\beta_{ij}}\right) \le \lambda \le \min_{\beta_{ij}<0}\left(-\frac{\alpha_{ij}}{\beta_{ij}}\right)$, hence the corresponding interval for the parameter λ $-\frac{2}{3} \le \lambda \le 0$, and the value of objective function

$Z = 2200 + 560\lambda$.

Step 5: In order to obtain the optimal solution, it is necessary to include field (3.2.) in the transportation program, following the MODI method rules through which the quantity of 80kg of fish is to be transported. Thus, a new improved solution is obtained, wherein the parameter λ can vary within the interval $0 \le \lambda \le \frac{1}{4}$, and the objective function has the value of $Z = 2200 + 400\lambda$.

The procedure is repeated, so for the values fulfilling the condition, it is necessary to use the field (3.1.) through which the quantity of 60kg of fish is to be transported. By repeating the steps described the optimal solution shown in Table 2 is obtained.

Table 2.

	P1	P2	P3	Capacity
S1	10 / 0	12 / **100** / -2	20 / 0	100
S2	4 / 8	2 / **0** / 0	4 / **120** / 4	120
S3	8 / **60** / -6	8 / **20** / 0	10 / 6	80
Demand	60	120	120	300

The interval of the parameter λ values corresponding to the optimal solution from Table 2 is $\frac{1}{4} \le \lambda \le 3$, and the value of the objective function $Z = 2320 - 80\lambda$.

5. CONCLUSION

The parametric programming is a specific method of mathematical programming aimed at solving optimization problems, with values depending on one or multiple parameters, which by its nature, has a very broad application in solving numerous economic problems.

By using the parametric approach to the transportation problem, the company management is able to determine the behaviour of the system if the parameters change within the given limits, as well as to set parameters so that, with the optimal solution determined, the greatest possible success is achieved, without significant distortion of the structure of the optimal solution.

6. REFERENCES

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