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FORMATION OF DIFFERENTIAL EQUATION DURING THE PROCESS OF MOTION OF TOWER-CRANE

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ABSTRACT
In this paper is reflected the analytical manner of motion of Tower crane. Simplifying the complicated processes of motion of crane will form a model of equivalent system of tower crane which comes from real model of tower crane. Tower crane have great importance in construction for this reason installation of tower crane must be so precise because crane must be in equilibrium during the work. The analyze of system with 2 DOF of motion is less exact than system with 3 DOF but gives good results and base for examining swings during the work of cranes. With sophisticated mathematical methods can be calculated their displacement and deflection in a way not to exceed limited values.
Keywords: Tower crane, mechanism for lifting, deflection, and rope

1. INTRODUCTION
Motion of revolving Tower crane is presented with equivalent system with 2 or 3 DOF (degree-of-freedom) of movement from method of dynamic principles. In these cases torque of motion can be taken as constant magnitude, influence of inertial forces of revolving masses of hoisting mechanism is not considered for system with 2 DOF. For motion system with 3 DOF considering influence of inertial forces of revolving masses for hoisting mechanism. Torque which is transferred by the engine with the mechanism for lifting at system with 3 DOF (degree-of-freedom) does not take constant magnitude. This moment is calculated with following expression:

$$M_{ni} = M_{nio} + h_i \dot{\varphi} \qquad (1)$$

M_{nio} – Torque of engine when starts, in the beginning still have n=0

h_i – Slope factor in managing curve

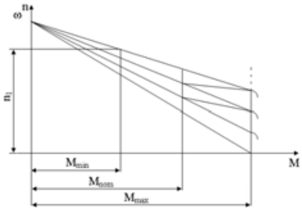

Figure 1. Graphical presentation of the issuance process of asynchronous engine by means of resistor with contact rings

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2. FUNDAMENTAL EQUATION FOR THE MECHANISM OF SLACK ROPE

. A lifted object does not usually participate in any horizontal motion because the rope only takes tension force. For rope system design, only vertical motion of the crane system is considered. Furthermore, the crane system with lifted load along vertical direction can be simplified as 2DOF . The fundamental equilibrium dynamic equations of crane system with lifted load can be expressed in 2DOF dynamic static;

$$m_1 \cdot \ddot{x}_1 + [c_1 \cdot (\dot{x}_1, x_1) - c_2 \cdot (x_1, x_2, \dot{x}_1, \dot{x}_2)] + [F_1 \cdot (x_1) - F_2 \cdot (x_1, x_2)] = -m_1 \cdot \ddot{x}_g \quad \text{(2)}$$

$$m_1 \cdot \ddot{x}_2 + c_2 \cdot (x_2, x_2, \dot{x}_2, \dot{x}_2) - F_2 \cdot (x_2, x_2) = -m_2 \cdot \ddot{x}_g \quad \text{(3)}$$

The masses of the crane system m_1 and lifted load m_2 are simply obtained by their weights divided by gravity acceleration g . The damping forces for crane c_1 and lifted load of rope c_2 can be defined as the function of the displacements and velocities as follows;

$$c_1(\dot{x}_1) = 2\xi_1\omega_1 \dot{x}_1, c_2(x_1, x_2, \dot{x}_1, \dot{x}_2) = \begin{Bmatrix} 2\xi_2\omega_2(\dot{x}_2 - \dot{x}_1) & if\,(x_2 - x_1) \geq -xs \\ 0 & otherwise \end{Bmatrix} \quad \text{(4)}$$

The ξ_1 and ξ_2 are the critical damping coefficient of the crane system and lifted rope, respectively .

3. MULTIPLE DEGREE OF FREEDOM SYSTEM OF TOWER CRANE

This is the easiest case of differential equation. The sketch of tower crane is presented below in the Figure 2.

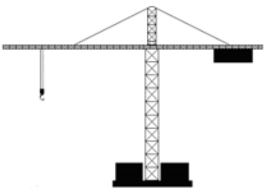

Figure 2. Tower crane with counter weight which is upper part of the crane

We examine the motion of tower crane for first period of motion, presenting equivalent system with three masses respectively with 3 DOF of motion, in this case counter weight is in the lower part respectively on the base. From the real sketch of tower crane we built equivalent sketch which is presented in figure 3.

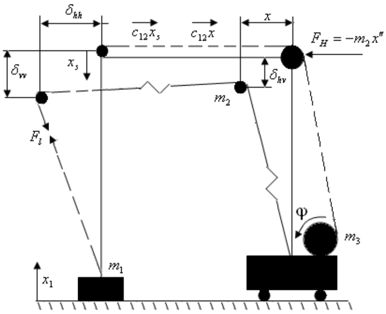

Figure 3 Equivalent model of Tower Crane rolling of system with multiple DOF.

m_1 - mass of load and reduced mass of vertical oscillation

m_2 - reduced mass of crane that is concentrated on top of tower

m_3 - revolving mass of mechanism for lifting placed in the axis of tumbler.

The torque of engine for hoisting accelerate tumbler and buckle rope which was free.

The process of hoisting start when the rope is buckled but is not loaded $t = 0$ is considered that tumbler is $\varphi = 0$. When the force on the rope is equal with the hoisted load, in this moment first period of motion is finished. In the first period of motion participate mass of mechanism for lifting and the mass of carrier construction which besides horizontal displacement they do vertical displacement as well.:

$$x_s = \delta_{vv} \cdot F_l + \delta_{vh} \cdot F_H \quad (5)$$

$$x = \delta_{hv} \cdot F_l + \delta_{hh} \cdot F_H \quad (6)$$

Impact coefficient of displacement:

$$\delta_{hv} = \delta_{vh} \quad (7)$$

F_l - Vertical force on the rope which act on top of wing

F_H - Horizontal force that act on the top of tower

$$F_l = \frac{\delta_{hh}}{\delta_{vv} \cdot \delta_{hh} - \delta_{hv}^2} x_s - \frac{\delta_{hv}}{\delta_{vv} \cdot \delta_{hh} - \delta_{hv}^2} x, F_H = -\frac{\delta_{hv}}{\delta_{vv} \cdot \delta_{hh} - \delta_{hv}^2} x_s - \frac{\delta_{hh}}{\delta_{vv} \cdot \delta_{hh} - \delta_{hv}^2} x \quad (8)$$

Hence;

$$F_H = -m_2 \cdot \ddot{x} \quad (9)$$

Relationship between impact coefficients δ_{ik} substituted with elastic constant C_{ik}, from above equation;

$$F_l = c_{22} \cdot x_s + c_{12} \cdot x, F_H = c_{12} \cdot x_s + c_{11} \cdot x \quad (10)$$

From equilibrium of forces is gained differential equation for motion of mass m_3 which in the first period of motion;

$$I_T \cdot \ddot{\varphi} + F_l \cdot r = M_n \quad (11)$$

I_T - Moment of inertial of revolve masses of mechanism for lifting concentrated in tumbler axis.

r - Radius of tumbler

Force in the rope;

$$F_l = c_l \cdot z \quad (12)$$

z - Extension of the rope that is expressed in relation;

$$z = (r \cdot \varphi - x_s) \quad (13)$$

$$F_l = c_l \cdot (r\varphi - x_s) \quad (14)$$

From the (8) equations can be found;

$$x_s = F_l / c_{22} - c_{12} / c_{22} x$$

If we substitute with above equations. The force in the rope is;

$$F_l = c_l \cdot r \cdot \varphi - \frac{c_l}{c_{12}} F_l + \frac{c_l \cdot c_{12}}{c_{22}} x,$$

When we regulate above equation can be written;

$$F_l = r \frac{c_l \cdot c_{22}}{c_l + c_{22}} \varphi + \frac{c_l \cdot c_{12}}{c_l + c_{22}} x \quad (15)$$

$$X_s = \frac{c_l \cdot c_r}{c_l + c_{22}} \varphi - \frac{c_{12}}{c_l + c_{22}} x \quad (16)$$

Knowing that nominal torque of electromotive is;

$$M_n = M_{no} - h_i \dot{\varphi} \quad (17)$$

Hence, differential equation of motion can be written;

$$m_2 \cdot \ddot{x} + c_{12} \cdot x_s + c_{11} \cdot x = 0, I_T \cdot \ddot{\varphi} + h_i \cdot \dot{\varphi} + F_l \cdot r - M_{no} = 0 \quad (18)$$

If we substitute equation for F_l and X_s, differential equation gain this form;

$$m_2 \cdot \ddot{x} + r \frac{c_l \cdot c_{12}}{c_l + c_{22}} \varphi - \frac{c_{12}^2}{c_l + c_{22}} x + c_{11} \cdot x = 0 \quad (19)$$

When we fix differential equation can be seen as following;

$$m_2 \cdot \ddot{x} + \frac{c_l \cdot c_{11} + c_{12} \cdot c_{11} - c_{12}^2}{c_l + c_{22}} x + r \frac{c_l \cdot c_{12}}{c_l + c_{22}} \varphi = 0, I_T \cdot \ddot{\varphi} + h_i \cdot \dot{\varphi} + r^2 \frac{c_l \cdot c_{22}}{c_l + c_{22}} \varphi + r \frac{c_l \cdot c_{12}}{c_l + c_{12}} x - M_{no} = 0 \quad (20)$$

Differential equations are solved applying the initial conditions;

$$t = 0,\ x = 0,\ \varphi = 0,\ \dot{x} = 0,\ \dot{\varphi} = 0,\ \ddot{x} = 0,\ \ddot{\varphi} = 0$$

From the equation (17) can be gained horizontal displacement x and angular displacement φ where the replacement according to the theory of oscillation.

$$x = A \cdot \sin \cdot (\omega t - \alpha)$$
$$x = A \cdot \sin \omega t \cdot \cos \alpha - A \cdot \cos \omega t \cdot \sin \alpha \quad \ldots\ldots (21)$$
$$\varphi = B \sin(\omega t - \alpha)$$
$$\varphi = B \cdot \sin \omega t \cdot \cos \alpha - B \cdot \cos \omega t \cdot \sin \alpha$$

After substituting of constants;

$$x = A_1 \sin \omega t - A_2 \cos \omega t \quad \ldots\ldots (22)$$
$$\varphi = B_1 \sin \omega t - B_2 \cos \omega t$$

After first derivate:

$$\dot{x} = \omega A_1 \cos \omega t - \omega A_2 \sin \omega t, \dot{\varphi} = B_1 \omega \cos \omega t - \omega B_2 \sin \omega t \quad \ldots\ldots (23)$$

Whereas, after second derivate:

$$\ddot{x} = -\omega^2 A_1 \sin \omega t + \omega^2 B_1 \cos \omega t, \ddot{\varphi} = -\omega^2 A_2 \cos \omega t + \omega^2 B_2 \sin \omega t \quad \ldots\ldots (24)$$

Solution of equation particular;

$$\varphi_p = M_{no} / I_T \bullet k \text{，} \; k = I_T \cdot (c_l + c_{22}) / r^2 (c_l \cdot c_{22}) \quad \ldots\ldots (25)$$

Homogenous solution is gained from determinant of coefficients with circular frequency is calculated as following after substituting of equations

$$\square(\omega^2) = \begin{bmatrix} -m_2\omega^2 + \frac{c_l \cdot c_{11} + c_{22} \cdot c_{11} - c_{12}^2}{c_l + c_{22}} & r\frac{c_l \cdot c_{12}}{c_l + c_{22}} & 0 & 0 \\ 0 & 0 & m_2\omega^2 - \frac{c_l \cdot c_{11} - c_{11} \cdot c_{22} - c_{12}^2}{c_l + c_{22}} & -r\frac{c_l \cdot c_{11}}{c_l + c_{22}} \\ -r\frac{c_l \cdot c_{12}}{c_l + c_{22}} & r^2\frac{c_l \cdot c_{12}}{c_l + c_{22}} - J_T\omega^2 & 0 & h\omega \\ 0 & h\omega & -r\frac{c_l \cdot c_{12}}{c_l + c_{22}} & J_T\omega^2 - r^2\frac{c_l \cdot c_{12}}{c_l + c_{22}} \end{bmatrix} \quad \ldots\ldots (26)$$

General solution of differential equations (17) can be expressed;

$$x = x_p + x_h$$
$$x = A_1 \sin \omega t + B_1 \cos \omega t + A_2 \sin \omega t - B_2 \cos \omega t$$
$$\varphi = \varphi_h + \varphi_p \quad \ldots\ldots (27)$$
$$\varphi = \beta(A_1 \sin \omega t + B_1 \cos \omega t + A_2 \sin \omega t - B_2 \cos \omega t) + I_T \cdot (c_l + c_{22}) / r^2 \cdot c_l + c_{22} \cdot M_{no} / I_T$$

Integrated constants are gained from boundary conditions of motion.

3. CONCLUSION

In this paper is analyzed displacement of tower cranes of construction respectively concentrated masses on the top of tower, on the top of the wing and the revolving mass of mechanism for lifting. All this is done on purpose to have a great stability of crane in a way that the work of crane to be safe and without risks of breaking down and may cause disastrous consequences in that operating area.

Motion of crane is controlled by using a mathematical model which is based on theory of oscillation using differential equations of second order with constant coefficient which are difficult to solve, but gives approximate and satisfactory results. In this paper is done a great job and a scientific contribution connecting theory with practice.

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