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THE APLICABILITY OF SIMPLIFIED RELATIONS FOR SYMMETRICAL COMPONENTS

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ABSTRACT

Having the same system of phasorial measures ($\underline{Y}_1$, $\underline{Y}_2$ and $\underline{Y}_3$), considered the phasor angles being φ_1, φ_2 and φ_3 with $\varphi_1 = 0$, $\varphi_2 = \varphi$ and $\varphi_3 = 2\varphi$, is trying to bring as simple as the direct succession, inverse and homopolar components, which depends only by the Y_1 and φ angle.

The appreciation of the unsymmetrical state can be made only based by the symmetrical components, determined using the Stokvis-Fortescue theorem. This mention is important in the context of proliferation of the calculus relations, which improvise the unsymmetrical state characterization, including in terms of implemented relations in equipments like qualimeter or Power-meter. In consequence, the symmetrical components are rigorously calculated based on the next relation, which presents the disadvantage to be expressed in complex, existing the less programming environments designated to work in this plan:

$$\underline{Y}_d = \frac{1}{3}\cdot\left(\underline{Y}_1 + a\cdot\underline{Y}_2 + a^2\cdot\underline{Y}_3\right),\ \underline{Y}_i = \frac{1}{3}\cdot\left(\underline{Y}_1 + a^2\cdot\underline{Y}_2 + a\cdot\underline{Y}_3\right),\ \underline{Y}_h = \frac{1}{3}\cdot\left(\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3\right). \quad (1)$$

Keywords: symmetrical components, phasors system, direct, inverse and homopolar components.

1. THE ITERATIVE CALCULUS METHOD

Developing (1) through the explaining of the a and a^2 operators and identifying the arguments of the trigonometric functions such as the sums may be written in an iterative form, based on the same summing index, the following set of calculus relations for the symmetrical components is proposed:

$$Y_d = \frac{1}{3}\cdot\left\{\left[\sum_{k=1}^{3} Y_k\cdot cos\left(\varphi_k + \frac{2\pi}{3}\cdot(k-1)\right)\right]^2 + \left[\sum_{k=1}^{3} Y_k\cdot sin\left(\varphi_k + \frac{2\pi}{3}\cdot(k-1)\right)\right]^2\right\}^{1/2} \quad (2)$$

$$Y_i = \frac{1}{3}\cdot\left\{\left[\sum_{k=1}^{3} Y_k\cdot cos\left(\varphi_k + \frac{2\pi}{3}\cdot(4-k)\right)\right]^2 + \left[\sum_{k=1}^{3} Y_k\cdot sin\left(\varphi_k + \frac{2\pi}{3}\cdot(4-k)\right)\right]^2\right\}^{\frac{1}{2}} \quad (3)$$

$$Y_h = \frac{1}{3}\cdot\left[\left(\sum_{k=1}^{3} Y_k\cdot cos\,\varphi_k\right)^2 + \left(\sum_{k=1}^{3} Y_k\cdot sin\,\varphi_k\right)^2\right]^{\frac{1}{2}} \quad (4)$$

corresponding to the direct (positive sequence) Y_d, inverse (negative sequence) Y_i and homopolar (zero sequence) Yh components. If the variables Y_k, for k€ {1, 2, 3}, are amplitudes or effective values, the variables (Y_d, Y_i, Y_h) are resulting like amplitudes, respectively like effective values. It can be remarked that according to (2)-(4), the phases of the symmetrical components may be determined as well; the real and imaginary parts of the expressions appear in this order and are comprised between square brackets, in the relations (2) and (3), respectively between round brackets, in (4).
The relations set (2)-(4) represents a scalar, iterative, calculus basis for the symmetrical components, which guide to identical results as the Stokvis-Fortescue theorem [4].

2. SIMULATION AND ANALYSIS OF THE UNBALANCED STATE

The verification of the symmetrical components calculus relations on as large as possible range of the dissymmetry coefficient, was made in [4] through the variation of the phase φ between the successive phasors of the three phase system inside the interval $\varphi \in[-2\pi/3, 2\pi/3]$. This fact permitted the scalling of a large domain of unbalanced states, starting from the direct sequence system, established for $\varphi=-2\pi/3$, passing through the omopolar sequence one when $\varphi=0$ and arriving to the inverse sequence (negative) one, for which $\varphi=2\pi/3$, even if the phasors modulus was mentained equals.
Applying the same simulation method of the unbalanced states, the analytically identifying of the unbalanced state quantities and indicators is made further on together with the graphical representations of these ones. In addition, the range of the variable φ will be extended to a complete interval (2π), in order to cover all possible unbalanced states.
Consequently, the unbalanced state simulation method consists in the following steps:
- the phasor modulus are considered equals,

$$Y_1=Y_2=Y_3; \tag{5}$$

- the phases of the three phasors, expressed in comparison with the variable $\varphi\in[-2\pi/3, 4\pi/3]$ and considering the first phasor in the axis of the reference system, are given by the relations:

$$\varphi_1 = 0;\ \varphi_2 = \varphi;\ \varphi_3 = 2\varphi \tag{6}$$

so that the three-phase system will be symmetrical for $\varphi=\pm 2\pi/3$ and omopolar for $\varphi=0$.
Introducing in (2) the phase quantities corresponding to the hypothesis mathematically expressed through (5) and (6), the modulus of the direct component is obtained as follows:

$$Y_d = \frac{Y_1}{3}\sqrt{4\cos^2 x + 4\cos x + 1} \tag{7}$$

where the notation $x = \varphi + 2\pi/3$ has been used.
Making the possible restriction and explaining the modulus function, a parts defined function is obtained as following:

$$Y_d = \begin{cases} \frac{Y_1}{3}\left[2\cos\left(\varphi + \frac{2\pi}{3}\right) + 1\right], & pt.\ \varphi \in \left[-\frac{4\pi}{3}, 0\right]; \\ -\frac{Y_1}{3}\left[2\cos\left(\varphi + \frac{2\pi}{3}\right) + 1\right], & pt.\ \varphi \in \left(0, \frac{2\pi}{3}\right). \end{cases} \tag{8}$$

The method is analogously used for the inverse succession component, for which the initial relation is (3) and resulting a similar parts defined function, like the next one:

$$Y_i = \begin{cases} \frac{Y_1}{3}\left[2\cos\left(\varphi - \frac{2\pi}{3}\right) + 1\right], & pt.\ \varphi \in \left[0, \frac{4\pi}{3}\right]; \\ -\frac{Y_1}{3}\left[2\cos\left(\varphi - \frac{2\pi}{3}\right) + 1\right], & pt.\ \varphi \in \left(\frac{4\pi}{3}, 2\pi\right). \end{cases} \tag{9}$$

Finally, the next function was identified for the homopolar component:

$$Y_h = \begin{cases} \frac{Y_1}{3}(2\cos\varphi + 1), & pt.\ \varphi \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]; \\ -\frac{Y_1}{3}(2\cos\varphi + 1), & pt.\ \varphi \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right). \end{cases} \tag{10}$$

The three functions, expressed by the relations (8)-(10), are periodical with the period (2π); the graphical representations of these ones are given in figure 1 for the range of the independent variable $\varphi \in [-2\pi/3,\ 4\pi/3]$ and considering the phasor modulus $Y_1 = 100$.

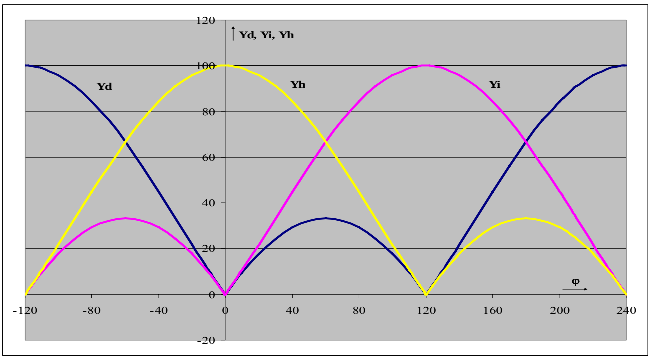

Figure 1. The symmetrical components Y_d, Y_i, Y_h graphical representations for a phasors system generated through h the method of the equal modulus and equal, consecutive, phases

The $Y_d(\varphi)$ function, given by (8), is continuous in the points where will be null:

$$\varphi \in \{2k\pi; 2\pi/3 + 2k\pi\},\ k \in Z \tag{11}$$

these ones representing minimum and angular points of the function.
The maximum of the function $Y_d(\varphi)$, $Y_{dM} = Y_1$ is given for $\varphi \in \{-2\pi/3 + 2k\pi\},\ k \in Z$ a local maximum point is existing as well, given by the relation $Y_{dMl}(\varphi \in \{\pi/3 + 2k\pi\}) = Y_1/3,\ k \in Z$. Similar considerations can be made for the function $Y_i(\varphi)$, correspondent to the inverse (negative) succession: the function will be annulled and presents minimum points (and angular) to the abscissa $\varphi \in \{2k\pi; 4\pi/3 + 2k\pi\},\ k \in Z$; the maximum of the function $Y_i(\varphi)$, $Y_{iM} = Y_1$ is given for the abscissa $\varphi \in \{2\pi/3 + 2k\pi\},\ k \in Z$; the local maximum points are given by the relation: $Y_{dMl}(\varphi \in \{-\pi/3 + 2k\pi\}) = Y_1/3,\ k \in Z$. In addition, the same characteristics for the $Y_h(\varphi)$ function, corresponding to the homopolar (zero succession) component are succinctly presented: the function is cancelled and presents minimum (and angular) points to the abscissa $\varphi \in \{-2\pi/3 + 2k\pi; 2\pi/3 + 2k\pi\},\ k \in Z$; the maximum of $Y_i(\varphi)$, $Y_{iM} = Y_1$ is given at the abscissa $\varphi \in \{2k\pi\},\ k \in Z$; the local maximum points: $Y_{hMl}(\varphi \in \{(2k+1)\pi\}) = Y_1/3,\ k \in Z$

3. THE DISSYMMETRY AND ASYMMETRY COEFICIENTS

The dissymmetry coefficient, named as well as negative unbalance factor (proposed notation - k_Y^-), is defined through the percentage ratio between the inverse (negative) succession Y_i and direct (positive) succession Y_d components, given by the relation:

$$K_{id\%} = \frac{Y_i}{Y_d} \cdot 100\ ,\ \%. \tag{12}$$

The asymmetry coefficient, named as well as zero unbalance factor (proposed notation - k_Y^0), is defined through the ratio between the homopolar (zero succession) and direct (positive) succession components, in percent:

$$K_{hd\%} = \frac{Y_h}{Y_d} \cdot 100\ ,\ \% \tag{13}$$

If, in the relation (12), who defines the dissymmetry coefficient, the determined expressions for the inverse succession (9) and direct succession (8) components are replaced according to the ranges of the corresponding functions and renouncing to percentage expression, the following relation for this factor is obtained:

$$K_{id} = \begin{cases} \dfrac{cos(\varphi - 2\pi/3) + 0,5}{cos(\varphi + 2\pi/3) + 0,5}, \; pt.\ \varphi \in \left(0, \dfrac{2\pi}{3}\right) \cup \left(\dfrac{4\pi}{3}, 2\pi\right); \\ -\dfrac{cos(\varphi - 2\pi/3) + 0,5}{cos(\varphi + 2\pi/3) + 0,5}, \; pt.\ \varphi \in \left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right]. \end{cases} \quad K_{hd} = \begin{cases} \dfrac{cos\,\varphi + 0,5}{cos(\varphi + 2\pi/3) + 0,5}, \; pt.\ \varphi \in \left(-\dfrac{2\pi}{3}, 0\right); \\ -\dfrac{cos\,\varphi + 0,5}{cos(\varphi + 2\pi/3) + 0,5}, \; pt.\ \varphi \in \left[0, \dfrac{2\pi}{3}\right]. \end{cases} \quad (14)$$

The graphical representations for both coefficients are presented in figure 2, for the same defining domain of the independent variable $\varphi \in [-2\pi/3,\ 4\pi/3]$. The both functions, $K_{id}(\varphi)$ şi $K_{hd}(\varphi)$, are not defined for the values $\varphi \in \{2k\pi; 2\pi/3 + 2k\pi\},\ k \in Z$.

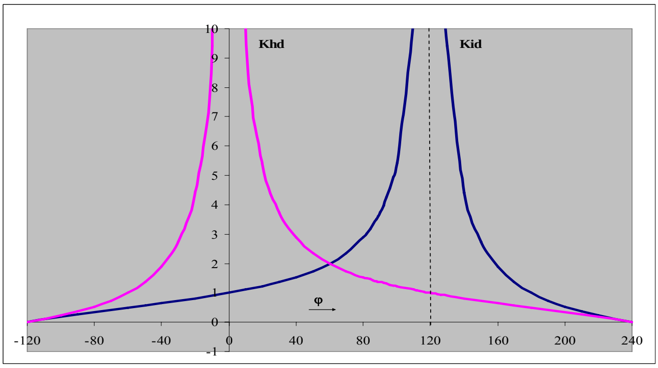

Figure 2. The dissymmetry and asymmetry coefficients K_{id}, K_{hd} graphical representations for a phasors system generated by the equal amplitudes and equal consecutive phase's method.

Restricting the definition domain to $\varphi \in [-2\pi/3,\ 4\pi/3]$, for which the graphical representations are made, it can be demonstrated that the function $K_{id}(\varphi)$ presents equal limits to the left and to the right, even it is not defined in origin:

$$\lim_{\varphi \to 0, \varphi < 0} K_{id} = \lim_{\varphi \to 0, \varphi > 0} K_{id} = 1 \qquad (15)$$

In the φ=2π/3 abscissa point, the function $K_{id}(\varphi)$ presents a vertical asymptote. The range of the function is $K_{id}(\varphi) \in [0,\ \infty)$, totally covered by the branch of the function from the right side of the asymptote, that is for $\varphi \in [2\pi/3,\ 4\pi/3]$, while the branch from the left side of the asymptote, for which the argument is placed in the range $\varphi \in [-2\pi/3,\ 2\pi/3]$, covers the range $K_{id}(\varphi) \in [0,1) \cup (1,\ \infty)$.

Regarding the function $K_{hd}(\varphi)$, expressed by (14), it has as a vertical asymptote the Y axis, with the equation φ=0 equation, i.e. at the abscissa for which the function $K_{id}(\varphi)$ is not defined, and for the abscissa where $K_{id}(\varphi)$ has the vertical asymptote φ=2π/3, where is not defined, it presents equal limits, to the left and to the right:

$$\lim_{\varphi \to 2\pi/3, \varphi < 2\pi/3} K_{hd} = \lim_{\varphi \to 2\pi/3, \varphi > 2\pi/3} K_{hd} = 1 \qquad (16)$$

The range of the function is $K_{hd}(\varphi) \in [0,\ \infty)$, totally covered by the branch from the left side of the asymptote, that is for $\varphi \in [-2\pi/3,\ 0)$, while the function branch from the right side of the asymptote, for which the argument is placed in the interval $\varphi \in (0, 4\pi/3]$, covers the interval $K_{hd}(\varphi) \in [0,1) \cup (1,\ \infty)$.

4. CONCLUSIONS

The utilization of the Stokvis-Fortescue theorem is essentially to characterize and analyse the unbalanced states. The derived scalar relations, like the iterative calculus ones, are very useful and practical for the analytical approach of the phasors unbalance systems. The phasors unbalanced systems generation is simple and efficient through the proposed method that is the method of the equal modulus and equal, consecutive, phases.

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