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MATHEMATICAL MODELING OF LINEAR AND NONLINEAR FLUID FLOW THE THROUGH THE RESERVOIR

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ABSTRACT
To determine the flow of liquid through the exit pipe of the reservoir is taken the mathematical model including quantity equation, local losses (valve) and cases when we have linear and nonlinear member in the flow equation. The aim of the paper is the indicating of error that is presented by linearization of nonlinear member and analytical and graphical illustration of the tolerated error. We will see that the time constants of the process in both cases show the accumulative ability of the reservoir water level as a ratio of volume flow respectively flow nominal diameters.
Key words: Equation of continuity, linear and nonlinear flow of liquid, diameter.

1. INTRODUCTION

In the largest number of technological processes the different tanks are widespread, i.e. reactors with their accumulation ability affect significantly in the dynamics of the whole process. In fig. 1 is shown the tank with all its elements. Following presentation, except basic purpose of determining the mathematical model of the object is intended to clarify the phenomenon of resistance to fluid flow, which on his part offers the local resistance in the valves and other elements.

Taking in Consideration of this problem is done with cases when there are linear and nonlinear member in the flow equation. Recognizing assumptions for homogeneous field flow, the flow is approximately stationary, on both sides prevailing atmospheric pressure, the velocity of the liquid level in the tank-reservoir is relatively small, and the continuity equation can be written for density ρ=const.

2. MATHEMATICAL MODELING OF LINEAR AND NONLINEAR FLUID FLOW THE THROUGH THE RESERVOIR

Starting from the material balance [1]:

$$\frac{dm}{d\tau} = \dot{m}_h - \dot{m}_d \tag{1}$$

or

$$A\frac{dh}{d\tau}=\dot{V}_h-\dot{V}_d \tag{2}$$

Considering the equation Bernoull's reached at the exit speed:

$$w=\varphi\sqrt{2gh} \tag{3}$$

Speed coefficient φ, flow coefficient μ and contraction coefficient ψ are:

$$\varphi=\frac{1}{\sqrt{1+\xi_v}}, \quad \mu=\varphi\cdot\psi \quad , \quad \psi=1 \tag{4}$$

Volume flow at the exit:

$$\dot{V}_d=\mu w_d A_d=\mu A_d\sqrt{2gh}=k_v\sqrt{h} \tag{5}$$

Where k_v depends on the flow coefficient and the surface of the transverse section of the choke valve:

$$k_v=\mu A_d\sqrt{2g} \tag{6}$$

By replacing equation (5) to eq. (2) achieved [2]:

$$A\frac{dh}{d\tau}=\dot{V}_h-k_v\sqrt{h} \tag{7}$$

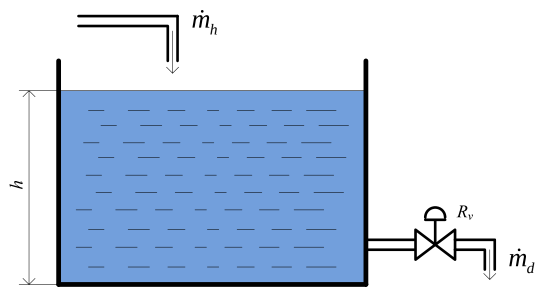

Figure 1. Reservoir

Mathematical models formed by linear and non-linear flow and optimal time achieving of the certain level of fluid in cases of nonlinear τjl and linear τl flow are expressed by following equations (table 1). If it is accepted that all conditions are fulfilled for linearization then for nominal regime can be written the corresponding data as the following table 1.

For linear conditions we have [3], [4]:

$$\dot{V}_d=\dot{V}_{dN}+\left(\frac{d\dot{V}_d}{dh}\right)_N\Delta h \tag{8}$$

or

$$\Delta\dot{V}_d=\left(\frac{d\dot{V}_d}{dh}\right)_N\Delta h \tag{9}$$

Considering the above expressions can be issued the resistance coefficient R_v which estimates the increase or flow:

$$\frac{1}{R_v}=\left(\frac{d\dot{V}_d}{dh}\right)_N=\frac{k_v}{2\sqrt{h_N}} \tag{10}$$

Table 1.

Nonlinear flow	Linear flow
$\dot{V}_d = \mu w_d A_d = \mu A_d \sqrt{2gh} = k_v \sqrt{h}$	$\dot{V}_d = \dfrac{1}{R_v} h$
$k_v = \mu A_d \sqrt{2g}$	$R_v = \dfrac{2h_N}{\dot{V}_{dN}}$
$A\dfrac{dh}{d\tau} = \dot{V}_h - k_v\sqrt{h}$	$A\dfrac{dh}{d\tau} = \dot{V}_h - \dfrac{1}{R_v} h$
$\tau_{jl} = -\dfrac{2A}{k_v}\left[\sqrt{h} - \sqrt{h_0} + \dfrac{\dot{V}_h}{k_v}\ln\left(\dfrac{\sqrt{h} - \dot{V}_h / k_v}{\sqrt{h_0} - \dot{V}_h / k_v}\right)\right]$	$\tau_l = -AR_v \ln\left[\dfrac{h - \dot{V}_h R_v}{h_0 - \dot{V}_h R_v}\right]$

3. ANALYSIS OF LINEAR AND NON-LINEAR EXIT FLOW AND LIQUID LEVEL

In view of the upper expressions, by means of the simulations respectively the diagrams which are presented in continuity, it is analyzed the reservoir of linear and non-linear exit flow. The liquid – water flow passed through the reservoir with volume flow shown in following figures, depending of diameters, liquid level, coefficient k_v, resistance coefficient R_v=0.001, 0.002 ... 0.005; ξv=3, h_0=0.1 m; $\dot{V}_h = 0.001$ m^3; D_h=0.01 m; D_d=0.01, 0.02...0.05 m; D=0.5,0.6...1.0 m;

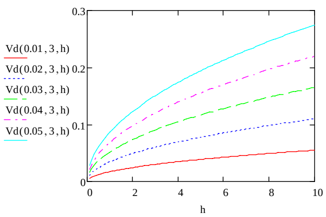

Figure 2. Nonlinear volume flow for D_d=0.01, 0.02...0.05 m; ξ_v=3; h=0.1, 0.2 ...10m

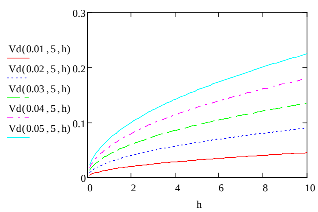

Figure 3. Nonlinear volume flow for D_d=0.01, 0.02...0.05 m; ξ_v=5; h=0.1, 0.2 ...10m

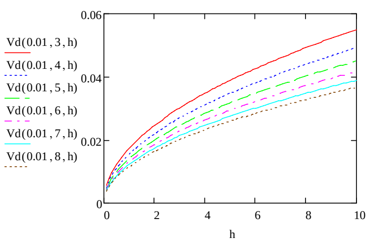

Figure 4. Nonlinear volume flow for D_d=0.01m; ξ_v=3, 4...8, h=0.1, 0.2 ...10m

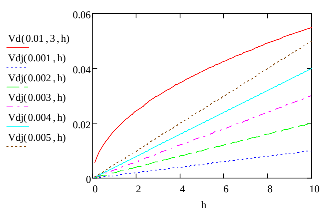

Figure 5. Linear and nonlinear volume flow for D_d=0.01 m; ξ_v=3; R_v=0.001, 0.002...0.005; h=0.1, 0.2 ...10m

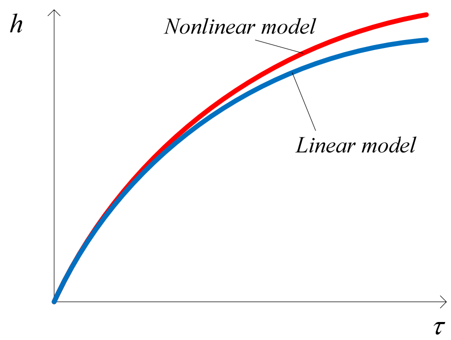

Figure 6. Liquid level changing for linear and nonlinear regime of flow

4. CONCLUSION

Graphic interpretation of the above expressions is given by curve 1 and 2 in figure. 6. Mathematical models, that above are presented, describe the dynamic of exit flow of reservoir. The above expressions indicate transmission ratios as measures, energy and impulse and their impact on the flow of fluid. Even if in practical situations avoid the use of square root equation-for linearization of flow , in the above figures noted visible changes made when we have nonlinear flow that means the mistakes must be intolerable as negative consequences.Achieved results consist in exactness and verity of practical side and also to secure the control flow by passing through the reservoir.

5. REFERENCES

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