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THE METHODOLOGY OF CONTROLLING MANUFACTURING PROCESSES WITH THE USE OF MULTIVARIATE STATISTICAL PROCESS CONTROL TOOLS

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ABSTRACT
The paper deals with the problem of controlling manufacturing processes with the use of statistical methods. In the article a comparison between traditional (univariate) and multivariate (taking into account many variables and relationships between them) statistical process control was made. Advantages and disadvantages of both approaches were pointed out as an introduction to the methodology of implementing multivariate statistical process control (MSPC) developed by an author. In the next part of the paper the author introduces consecutive steps of his methodology. This methodology is divided into three main stages: planning stage, stage of examining stability and capability of the process and monitoring stage. It is based solely on the use of Hotelling control charts and multivariate capability indices. At the end of the paper some conclusions concerning multivariate statistical process control use were made.
Keywords: MSPC, Hotelling charts

1. INTRODUCTION
Statistical process control (SPC) consists in controlling a stability and capability of a process with the use of statistical methods and tools [1]. The most popular representatives of this group are control charts (especially Shewhart control charts) and capability indices (Cp and Cpk). but they take into account only one variable at the same time – they are univariate. The counterpart of SPC in the multivariate domain is Multivariate statistical process control (MSPC). It consists in monitoring simultaneously a group of variables with the use of one aggregated statistic – the most frequently Hotelling T^2 statistic [2,3,4,5]. The most popular MSPC tools are T^2 Hotelling charts and multivariate capability indices. Proper usage of SPC as well as MSPC tools is leading to continuous improvement of processes. It is described for example in [6].

2. SPC AND MSPC TOOLS – ADVANTAGES AND DISADVANTAGES
SPC as well as MSPC tools have many advantages and disadvantages connected among others with their ease of implementation, complexity of mathematical apparatus, ease of interpretation and popularity. The comparison of univariate and multivariate control charts (which are the most frequently used MSPC tools) was summarized in Table 1.
On the basis of Table 1 it is easy to conclude that univariate control charts are very well-known in industrial environment. Their ease of use, implementation and interpretation make them very popular in practice. Besides the literature describing univariate SPC is very widespread. Unfortunately, traditional (Shewhart) control charts are not suitable for monitoring many variables mainly because of separate significance level for any chart, not taking into account a relationship between variables and

"organizational mess" connected with using separate control charts for every important process/product variable.

Table 1. The comparison of univariate and multivariate control charts (own work on the basis of [7]).

Criterion	Univariate control charts	Multivariate control charts
Popularity of tool in industry	very popular	very rarely used
Ease of implementation	because of availability of many guides – easy, organizationally – quite easy	because of unavailability of guides and clear methodology – difficult, organizationally – very difficult
Need to use special software	not requirement but advisable	obligatory
Taking into account relationship between variables	do not take into account	take into account
Simultaneous monitoring of many variables	by using many univariate control charts – it is a big problem for an operator of the process	by using one aggregated multivariate statistic and monitoring it on one control chart sheet
Significance level	separate for every control chart, global – impossible to compute	one significance level for the whole control procedure
Interpretation of out-of-control signal	very easy and straightforward	interpretation is practically impossible without use of special methods
Existing of run rules	they exist and are widely used	they do not exist
Unit of controlled statistic	the controlled statistic is in unit of controlled variable	the controlled statistic has no unit
Ease of understanding the mechanism of functioning	very easy	quite difficult

Multivariate control charts let fill this gap by taking into account a relationship between variables (with the use of covariance matrix modeling it), monitoring only one statistic on the control chart sheet and using one global significance level for the whole control procedure. They are a very good alternative for univariate control charts in multivariate space despite they are very complicated, difficult to implement and interpret and require special software.
Unfortunately MSPC tools are not so popular in industrial practice. Research conducted by Wytyk [8] showed that among 97 leading production enterprises of Wielkopolska region in Poland only one uses multivariate methods. It can result from earlier specified disadvantages but also from the lack of clear methodology of implementing multivariate control charts and capability indices (like existing Chrysler_Ford_General Motors reference manual [9] concerning SPC in the univariate domain). That is probably why industrial practitioners choose better described and straightforward traditional Shewhart control charts.
In the next chapter a methodology of implementing and using multivariate statistical process control tools was introduced together with some methodological guidelines.

3. METHODOLOGY OF IMPLEMENTING MSPC

The methodology developed by the author consists of three stages:

- planning stage,
- examining the stability and capability of the process,
- process monitoring stage

The methodology was designed to consist of some consecutive easy to follow steps, be universal and elastic and be based on using multivariate control charts and multivariate capability indices. The general diagram of the methodology was shown in the Figure 1.

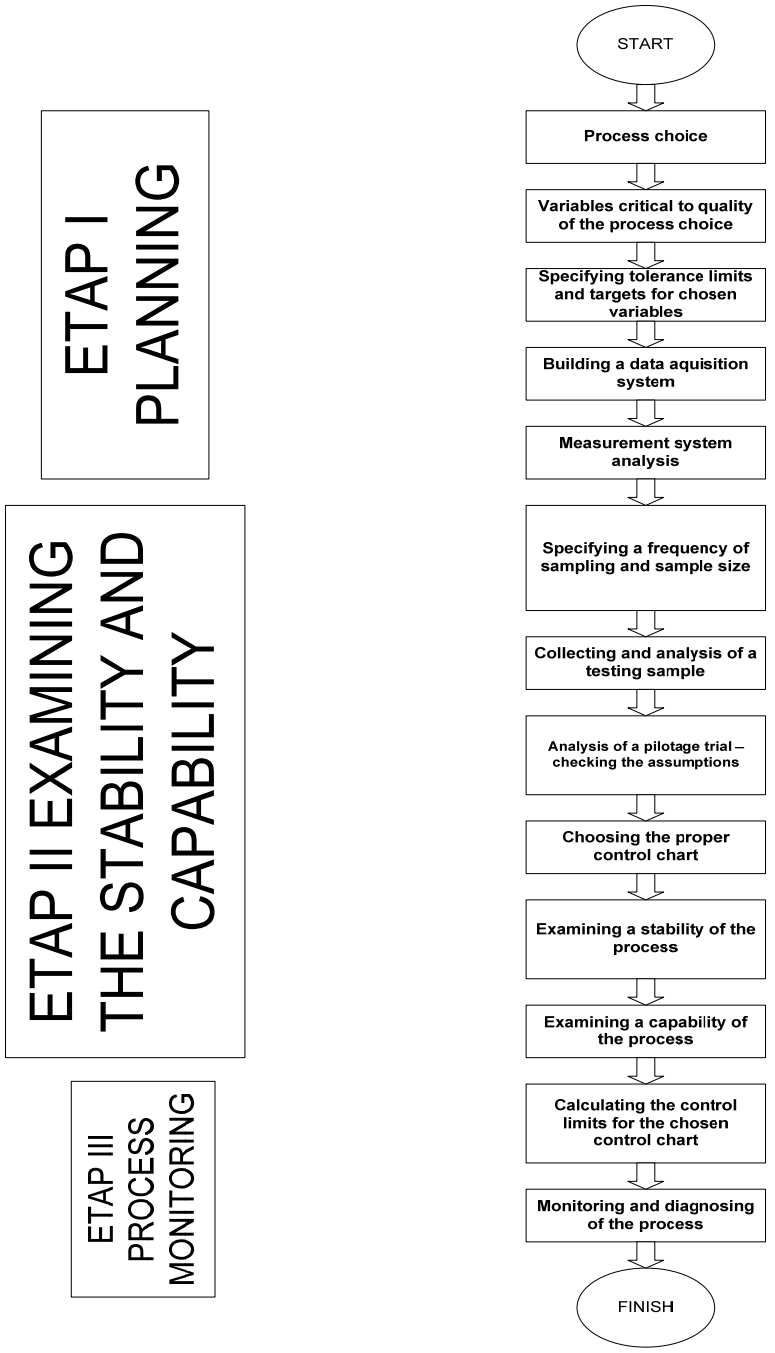

Figure 1. The methodology of implementing multivariate statistical process control tools (own work on the basis of [7])

The first stage of implementing MSPC is planning. It consists of all the activities connected with choosing the most suitable process to control and ensuring the appropriate environment to follow the control procedure using such sophisticated statistical tools. It contains among others specifying of the

main variables and requirements concerning them and something what distinguishes significantly multivariate procedure from univariate one: building a proper data acquisition system. It should ensure collecting a full record of data in one place and time to properly control the process. The last step in this stage is MSA. Measurement system should follow specified criteria [10].
The second stage of implementing MSPC is examining the stability and capability of the process. It consists mainly in collecting and analyzing a testing sample and discarding outliers (earlier finding out what led to their appearance) to obtain a "clean" pilotage trial. On the basis of it very important assumptions are checked (multivariate normal distribution of the data, independence of collected samples and non-singularity of the covariance matrix) and proper control chart is chosen. With the use of this control chart a stability of the process is verified and then multivariate capability indices are computed to find out if the process is capable to meet quality requirements. It is very important to notice that these indices should also be multivariate to take into account all important process variables. If the process is stable and capable, the control limits on multivariate control chart serve as an input to monitor and diagnose the process.
The third stage of implementing Multivariate statistical process control is monitoring and diagnosing the process. The main objective of this phase is keeping the process stable and capable. It is accomplished by observing the control chart sheet and looking for some non-random symptoms of variation originating from special causes. Characteristic for multivariate procedure is that out-of-control signal can be caused by one variable, many variables or relationship between variables. That is why the control procedure should be aided by methods of decomposing the T^2 statistic, e.g. MYT decomposition [4] to find out what is a source of the signal. It is one more thing which distinguishes SPC from MSPC.

4. CONCLUSIONS

As it was shown implementing MSPC tools is not an easy task for practitioners. Very sophisticated mathematical apparatus and some methodological aspects of implementation make them very difficult to utilize. However, in multivariate area multivariate control charts proved to be effective tool to monitor manufacturing processes. That is why the particular pressure should be put on encouraging quality engineers and management that they are worth using. It is also very important to simplify procedures connected with their usage..

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