

WEIGHTED GOAL PROGRAMMING OPTIMIZATION DIET MODEL

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ABSTRACT

The aim of this paper is to develop a goal programming optimization diet model subject to specific weighted model goals. Objective function in the model is designed in a way to minimize sum of the deviations percentages of the weighted goals with respect to food cost and energy density. The World Health Organization (WHO) standards for nutrients needs are incorporated in the goal programming model constraints. The food commodities used in the research are selected based on a survey of households in the capital of Bosnia and Herzegovina. Results are obtained in a form of optimal diet according to minimized weighted proportional deviations.

Keywords: goal programming, weighted goals, deviations percentages, diet, food cost

1. INTRODUCTION

Reference [1] develops optimization diet models for the reference man and the reference woman using linear programming optimization technique. Objective function in the model is food cost, while constraints are defined as daily needs for macronutrients and micronutrients in accordance with the standards of the World Health Organization (WHO) and National Academic Press (NAP).

Research [2] show that for the reference woman and the reference man it is possible to develop reliable goal programming mathematical model in order to make compromise between the food cost and micronutrients and macronutrients intake.

Incentives for research [3] was the question of whether diet with higher nutrient density and lower energy density, cost more. The study was conducted in France, based on a survey of national food consumption, and food costs are estimated based on the retail price. For this research, the participants were stratified by quartiles of energy costs. The survey targeted at assessing the relationship between energy, energy density, and nutrients. Energy-dense food is cheaper for consumers, but it is also low in nutrients.

The aim of study [4] was to establish a relationship between energy, nutrients and food costs using data for nutritional composition and food prices from the USDA database. The analysis results show that the highest energy density (kcal/100 g) has a group of fats and oils, and the lowest a group of fruits. As for the unit price (\$/100 g), the highest price has a group of meat, poultry and fish, and the lowest group of fruits. The highest cost per serving (\$ / serving) has a group of meat, poultry and fish, and the lowest a group of fats and oils. Results also show that the group of grains and fats, oils have lower energy costs (\$/100 kcal) than groups of vegetables and fruits.

In study [5] the objective was to assess the relationship between diet quality and cost of low-income women between 20 and 55 years in four counties in California. The results showed that a diet with a low energy density has a higher energy cost and higher nutrient content. Nutrition with a higher

energy density and lower cost contains more fat and sugar, and less dietary fiber, vitamins A and C. Thus, this study showed that a healthier diet need more money. The aim of this paper is to develop weighted goal programming model with food cost and energy density as goals.

2. RESEARCH METHODOLOGY

The term "balanceddiet" means that a diet meet the right types and amounts of foods and drinks to supply nutrition and energy for maintaining body cells, tissues, and organs, and for support normal growth and development. On population level nutrient intake goals represent the population average intake that is consistent with the maintenance of health in a population. Health, in this context, is marked by a low prevalence of diet-related diseases in the population. To deal with the increasing public health problem - the growing epidemic of chronic diseases WHO made public health recommendations regarding prevention and the reduction of their impact.

Energy daily intake from macronutrients for the reference man is as follows: saturated fatty acids <10%, n6 polyunsaturated fatty acids 5-8%, n3 polyunsaturated fatty acids 1-2%, monounsaturated fatty acids 10-14%, total carbohydrate 50-70%, free sugars <10% and protein 10-15%.According to WHO definition the reference man is healthy male person aged 25 with an average mass of 65 kg and low active lifestyle.In models developed in this paper an additional constraintsare included that intake of raw vegetables and fruits must be at least 400 grams, total dietary fibre limits from 27 g - 40 g,and energy intake for the reference man of 2.590 kcal, as recommended by WHO. Micronutrients inadequate intake can cause serious health problems, such as anemia, rachitis in case of malnutrition, or toxic effect in case of excess intake. Because of that for each nutrient daily intake WHO recommendsRNI value(Recommended Nutrient Intake) and UL value (Upper Limit) .

In this study goal programming model was designed to meet all micronutrient WHO recommendations (vitamins A, D, E, K, B₆, B₉, B₁₂, C and minerals calcium, sodium, zinc and iron).

Table 1. showsmicronutrient requirementsforthe reference man.

Table 1. Micronutrient requirement

Nutrinet limits	RNI	UL
Vitamin A (µg)	300	600
Vitamin D (µg)	5	50
Vitamin E (mg)	15	1000
Vitamin K (µg)	65	-
Vitamin B6 (mg)	1.3	100
Vitamin B9 (µg)	400	1000
Vitamin B12 (µg)	2.4	1000
Vitamin C (mg)	45	1000
Ca (mg)	1000	3000
Na (mg)	1000	2000
Zn (mg)	7	45
Fe (mg)	8	45

Data of 55 most frequently consumed food commodities are used for this study. Prices of included food commodities are the market prices expressed in local currency Convertible Mark (KM).Dietary guideline should be flexible in means of adoption to specific dietary pattern preventing unbalanced diet, including undernutrition and overnutrition. Habits of consumptionof certain quantity of daily meal ought to be considered because variation in food quantity is followed by inadequate intake of energy. In last few decades energy intake increased by approximately 450 to 600 kcal per person per day.Public strategies must achieve the consumption of adequate quantities of good quality foods that makeup a healthy diet including energy density recommendations.

Mathematical models that compare quantity and quality of food with food cost and energy density could be useful in this strategy.

3. MATHEMATICAL MODEL AND RESULTS

Paper [1] describes linear programming local cost nutrition optimization model with minimization of the cost of the food as the objective function. Limitation of the model is that it is developed for daily needs for the reference man and thus doesn't include food variety consumption as a function of time as natural human need. To overcome this limitation of the model in this paper food variety consumption is included. In the model 1 in this paper objective function is defined by equation:

$$\text{Minimize } Z = \sum_{i=1}^{i=55} C_i x_i \quad \dots(1)$$

where:Z = total cost of food identified by model, x_i = decision variables representingmass of selected food commodity expressed in grams per day, C_i =cost of decision variable x_i expressed per mass unit of food commodity.

The objective function is subject to the following constraints:

$$\sum_{i=1}^{i=55} a_{ji}x_i \geq (RNI)_j \quad \dots(2)$$

$$\sum_{i=1}^{i=55} a_{ji}x_i \leq (UL)_j \quad \dots(3)$$

$$x_i \geq 0 \quad \dots(4)$$

where: j = double sided limits of micronutrients and macronutrients, a_{ji} = content of j^{th} micronutrient or macronutrient per mass unit of i^{th} food commodity.

To ensure some food variety consumption, next model, model 2, is developed where percentages of certain commodities inside its food commodity group is set as variable with limitations. The food is divided into 7 major groups according to modified Shermann scheme: 1. cereals, bread and pasta (including snacks); 2. sugar and sugar concentrates; 3. fat (including nuts); 4. meat, fish and eggs; 5. milk and dairy products; 6. fruits and 7. vegetables. Food commodities inside cereal, fruit and vegetable groups are defined with variable limitations, while the percentages of the food commodities in the following groups are not limited: fat; meat, fish and eggs; milk and dairy products group; and sugar and sugary concentrate group. The value of the objective function for the model 2 without limitations is 3,39 KM with food mass of 1.045,01 g. Binding constraints for this model are minimal requirements for sodium, calcium and vitamin D and maximal limits of vitamin A, diet fiber, vitamin B9 and constraints that energy intake coming from polysaccharide must be at least 50% and from n3 fat acid must be at least 1%. Apples in fruit group participate with 314,00 g or 99%. Energy density for this model is 247,85 kcal/100 g.

The value of the objective function for the model 2 with 20% limitation of participation of one food commodity within its group commodity is 3,97 KM. Binding constraints for this model are minimal requirements for sodium, calcium and vitamin K, and maximal limits of vitamin A, diet fiber, and constraints that energy intake coming from polysaccharide must be at least 50% and from n3 fat acid must be at least 1% and from monounsaturated fat acids must be at least 10%. Apples in fruit group participate with 57,60 g or 20%. This 20% limitation caused that in model solution oranges, tangerines, lemons and bananas are included and thus some variety of food intake goal is achieved.

It is interesting to analyze how this percentage limitations impact the value of the objective function (food cost). Figure 1. depicts that the food cost changes from 3,39 KM (case with no limitations) to 3,97 KM (case with 20% limitation of participation of one food commodity within its group commodity). Figure 2. shows that in the case of 20% limitation of participation of one food commodity within its group commodity objective function increases 16,42%. It can be seen that the variety of food is significantly increased for a small increase of food cost.

In order to satisfy requirements for the energy density diet model 3 is developed with objective function changed from food cost minimization to food mass maximization with the same constraints. For this model the value of the objective function is 2.861,40 g with food cost of 7,20 KM, resulting in energy density of 90,52 kcal/100 g. Model solution includes: rice (109,83 g), cornflakes (21,57 g), potato (1.455,37 g), sunflower oil (30,34 g), olive oil (3,89 g), walnut (14,80 g), canned sardines (69,96 g), milk 3,2% (218,52 g), milk 1% (537,05 g), mushrooms (360,53 g) and cabbage (39,47 g). Model 1 and model 3 have conflicting goals. In order to overcome these conflicting demands weighted goal programming model 4 is developed.

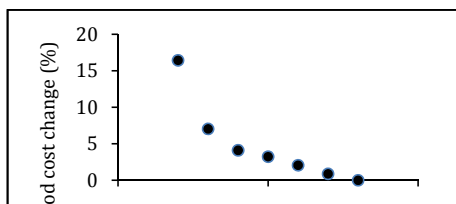


Figure 1. Percentage limits vs. food cost in KM

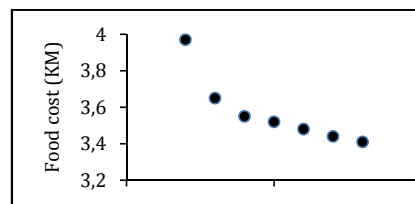


Figure 2. Percentage limits vs. food cost change in %

In the weighted goal programming model 4 food cost value of 4,5 KM and food mass of 1.500 grams are defined as goals, with food cost vs. food mass weight ratio of 2:1 including 20% limitations as described above. Since goals are measured in different units proportional deviations in objective function are used as presented in equation (5). Objective function in the model is defined as:

$$\text{Minimize } Z = \sum_{i=1}^{i=2} \left(w_i \frac{d_i^-}{g_i} + w_i \frac{d_i^+}{g_i} \right) \quad \dots(5)$$

where: d_i^- , d_i^+ are positive and negative deviational variables representing deviations from the i^{th} goal, i = number of goals and w_i = weights.

Equations (2) and (3) define UL and RNI model constraints regarding nutrient and energy needs as stated in model 1. The objective function is subject to the following constraint with respect to food mass, M :

$$\sum_{j=1}^{55} b_j \cdot x_j + (d_M^- - d_M^+) = M \quad \dots(6)$$

where: b_j = mass in grams for the j^{th} food commodity, d_M^- , d_M^+ = positive and negative deviations from food mass goal and M = food mass goal.

The objective function is also subject to the following constraint with respect to the food cost:

$$\sum_{j=1}^{55} c_j \cdot x_j + (d_c^- - d_c^+) = C \quad \dots(7)$$

where: c_j = cost per gram of j^{th} food commodity, d_c^- , d_c^+ = positive and negative deviations from the food cost goal and C = food cost goal.

Results of the optimization model show that food cost goal is completely achieved, while food mass goal is underachieved with negative deviational variable $d_M^- = 37,40$ grams.

4. CONCLUSION

Linear programming model with food cost minimization as the objective function shows that with inclusion of food variety by limiting food commodities in one food group to 20%, minimal food cost increases about 16%. Linear programming model with energy density maximization, as the objective function, increases food cost by around 80%. Third goal programming model shows that it is possible to develop reliable weighted goal programming mathematical model in order to make compromise between the food cost and energy density while satisfying micronutrients and macronutrients intake according to WHO recommendations. Food cost goal is set according to local food prices. In order to avoid infeasible solution in the weighted goal programming model as a result of prescribed goal values two linear programming models with objective function of food cost minimization and energy density maximization are solved. Solutions of these models determine limit values of the goals in the weighted goal programming model. Defining goals and weights with respect to these limits in the goal programming model is up to a decision maker.

5. REFERENCES

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