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NEWSVENDOR EXTENSIONS WITH BACKORDER OPTION

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ABSTRACT
In this paper two new newsvendor modelextensions are developed. These two extensions allow modeling of the situationswhere there are certain percentage of customers who are willing to wait for the next regular order.
The first model assumes that it is possible to identify customers who are willing to wait for the next regular orderat the begining of planning period, while the second modelassumes backorderoptionat time when stock out occurs.
Keywords: inventory, extensions newsvendor model, newsvendor model with backorder option

1. INTRODUCTION
There are a number of logistics systems and cases in which orders may be placed only at certain times. Ordering dates of these systems are known in advance, because they are imposed by some limitations of the logistics system itself, or are due to the specific characteristics of the products kept on stock. Objective of inventory control, in these systems, is to determine the number of orders that will be sufficient to cover demand between two consecutive orders. Ordering too little, in these systems and cases, means that a demand can not be satisfied, and additional costs will occur, such as additional order costs, lost sales costs, reputation and business credibility loss, future contracts and sales loss, etc. Ordering too much, means the occurrence of excessiveinventory levels, that generate costs whose values depends on the nature of the product.Modeling of described inventory control problems can be successfully done using newsvendor model and its extensions.

2. LITERATURE REVIEW
Newsvendor problem has been presentin the literaturefor over 100 years [1], and newsvendor model that solves newsvendor problem is one of the most famous models in the operating management and operational research, in general [2]. This model, even if introduced in the middle of the last century [3], still attracts the attention of a large number of authors in recent years [4].
Widespread of newsvendor problems and newsvendor modelpopularity have resulted in numerous articles dealing exclusively with taxonomy of newsvendor modelextensions, or articles exclusively dealing only with literature reviews in this filed [4,5,6,7].Applicability of newsvendor model is manifold. Inventories in the food and the clothing industry are often modeled using newsvendor model [8]. Newsvendor model is also used in modelling and solving problems in the production capacity management, and in service industries, such as airline and hotel reservations [9,10]. As the lifetime of

the product continues to shorten, the importance of the newsvendor models grows, so many newsvendor modelextensions are developed in the last few years [4,11,12].

3. NEWSVENDOR EXTENSION WITH BACKORDER OPTION

For derivation of newsvendor extension with backored option, and for the definition of classic newsvendor model and other mentioned extensions, folowing notation have been used:
x – demand, random variable; $f(x)$ – probability density functionof$x$; $F(x)$ – cumulative distribution function of x; P – selling price per unit; C – buying price per unit; V–salvage value per unit; S – shortage penalty cost per unit; b – percent of demand that can be satisfied from the next regular order, in case of backorder;Q – ordering quantity, decision variable.

Optimal ordering quantity for classic newsvendor model can be calculated by:

$$Q^* = F^{-1}\left(\frac{P+S-C}{P+S-V}\right). \qquad \dots(1)$$

3.1. Newsvendor model withbackorder optiontriggered when stock out occurs

Profit in one planning period is given by:

$$\pi = \begin{cases} (P-C)Q + (P-C)(x-Q)b - (1-b)S(x-Q), & \text{if} \geq Q \\ (P-C)x - (C-V)(Q-x), & \text{if} < Q \end{cases}, \qquad ...(2)$$

where:$(P-C)Q$– profit when$Q$ units are sold if$x \geq Q$; $(P-C)(x-Q)b$– profit when$(x-Q)b$units are soldif$x \geq Q$; $(1-b)S(x-Q)$– total shortage cost if$x \geq Q$; $(P-C)x$– profit when$x$units are soldif$x < Q$,$(C-V)(Q-x)$– total excess inventory cost if$x < Q$.
Expected profit per planning period is given by :

$$E(\pi) =$$
$$= [P - C - (P-C)b + (1-b)S]\int_Q^{+\infty} Qf(x)dx +$$
$$+[(P-C)b - (1-b)S]\int_Q^{+\infty} xf(x)dx +$$
$$+(P-V)\int_0^Q xf(x)dx - (C-V)\int_0^Q Qf(x)dx. \qquad ...(3)$$

First derivative of expected profit function can be found using Leibniz's rule:

$$\frac{\partial E(\pi)}{\partial Q} = (P-C+S)[1-F(Q)] - (P-C+S)b[1-F(Q)] - (C-V)F(Q). \qquad ...(4)$$

If first derivative of expected profit function is set to zero, then follows:

$$F(Q_B^*) = \frac{(1-b)(P+S-C)}{(1-b)(P+S)+Cb-V}, \qquad ...(5)$$

where:$Q_B^*$is optimal ordering quantity, in case that expected profit function is concave, which can be proved by calculation of its second derivative. Explicit expression for calculation of optimal ordering quantityQ_B^*follows from (5):

$$Q_B^* = F^{-1}\left(\frac{(1-b)(P+S-C)}{(1-b)(P+S)+Cb-V}\right). \qquad ...(6)$$

If expressions (1) and (5) for optimal ordering quantities are compared, than can be proved that $Q_B^* \leq Q^*$, in case these relations stand$V < C < P + S$.

3.2. Newsvendor model with backorder optiontriggered at the begining of planning period

Profit in one planning period is given by:

$$\pi = \begin{cases} (P-C)Q(1-b) + (P-C)Qb - (1-b)S(x-Q), & \text{if } x \geq Q \\ (P-C)x - (C-V)(Q-x), & \text{if } x < Q \end{cases}, \quad ...(7)$$

where: $(P-C)Q(1-b)$ – profit when$Q(1-b)$units are soldif$x \geq Q$; $(P-C)Qb$ – profit whenQbunits are soldif$x \geq Q$; $(1-b)S(x-Q)$ – total shortage cost if$x \geq Q$; $(P-C)x(1-b)$ – profit when$x(1-b)$units are soldif$x < Q$; $(P-C)xb$ – profit whenxbunits are soldif $x < Q$; $(C-V)(Q-x)$– total excess inventory cost if $x < Q$.

First derivative of expected profit function can also be found using Leibniz's rule:

$$\frac{\partial E(\pi)}{\partial Q} = (P - C + (1-b)S)[1 - F(Q)] - (C-V)F(Q). \quad ...(8)$$

If first derivative of expected profit function is set to zero, then follows:

$$F(Q_B^{**}) = \frac{(P-C)+(1-b)S}{P-V+(1-b)S}, \quad ...(9)$$

where:$Q_B^{**}$is optimal ordering quantity, in case that expected profit function is concave, which can be proved by calculation of its second derivative. Explicit expression for calculation of optimal ordering quantityQ_B^{**}follows from (10):

$$Q_B^{**} = F^{-1}\left(\frac{(P-C)+(1-b)S}{P-V+(1-b)S}\right). \quad ...(10)$$

If expressions (1) and (5) for optimal ordering quantities are compared with expression (9), than can be proved that $Q_B^{**} \leq Q_B^{*} \leq Q^{*}$, in case these relations stand$V < C < P + S$.

4. CONCLUSION

Derived extensions of newsvendor model with backorder option are models that allow achieving the same service level with lower inventory levels in the case when there is a percentage of customers willing to wait for the next regular order. In situations where there are no buyers who are willing to wait for the next order, than these situations can be successfully modelled with developed extensions, where developed extensions become a classic newsvendor model. In situations where there are buyers who are willing to wait for the next regular order, newsvendor model with backorder option, that is triggered after the stock out, is more efficient than the classic newsvendor model, and the newsvendor model with backorder option, that is triggered at the beginning of the planning period, is more efficient than both models. In situations where all customers are willing to wait for the next regular order, both extensions of the newsvendor model developed in this paper, will give the same result. Developed extensions could be even more efficient if they could enable modelling the percentage of customers who are not willing to wait for the next regular order, but are willing to wait for emergency order.

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