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FINITE VOLUME APPROACH TO FLOW OF HIGH CONCENTRATION FLY ASH SLURRY

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ABSTRACT

Development of the hydrodynamic field for fly ash slurry as a viscoplastic fluid is solved numerically using finite volume method for the solution of the mass and momentum equations in integral form. To illustrate the applicability and efficiency of the method the vertical flow through a straight constant radius pipe as well as through an axisymmetric sudden expansion is presented. The solutions obtained for the examples are valid for a wide range of Reynolds and Yield numbers and are compared with other existing solutions based on reduced forms of the governing equations. This work demonstrates that the finite volume method can be successfully employed to obtain solution for wide range viscoplastic materials with emphasis on fly ash slurry flow.

Keywords: fly ash, rheology, pressure losses, finite volume, viscoplastic flow, Bingham plastic.

1. INTRODUCTION

In many industrial applications fluids exhibiting certain minimum stress level that has to be exceeded before the fluid flows are used. Such fluids cannot sustain a velocity gradient unless the magnitude of the local shear stress is higher than the yield stress τ_0 and are called viscoplastic or "yield stress" fluids (cement, fly and bottom ash slurries, grease, etc). The rheological behavior of a number of these fluids can be described by the constitutive equation for a Bingham fluid. Although these fluids are of importance in many industries, the literature on the internal flow and thermal behavior of Bingham fluids is limited. In an early study, Chen et al. [3] employed an integral boundary layer formulation to calculate the laminar flow of a Bingham fluid in the entrance region of a straight circular pipe. Vradis et al. [6] numerically investigated simultaneously developing velocity and thermal fields of a Bingham fluid in the entrance region of circular pipes. Knežević and Kolonja [5] analyzed the change in flow and pressure, in relation to the solid concentration change in the mixture of fly ash slurries.

2. MATHEMATICAL MODEL

By applying the principles of conservation of mass and momentum to an arbitrary volume V bounded by the surface *S*, the mass balance equation and momentum balance equation are obtained:

$$\frac{d}{dt}\int_V \rho dV + \int_S \rho \mathbf{v} \cdot d\mathbf{s} = 0 \tag{1}$$

$$\frac{d}{dt}\int_V \rho \mathbf{v} dV + \int_S \rho \mathbf{v} \otimes \mathbf{v} \cdot d\mathbf{s} = \int_S \mathbf{T} \cdot d\mathbf{s} + \int_V \mathbf{f}_b dV , \tag{2}$$

where *t* is the time, ρ is the density, $\mathbf{v}$ is the fluid velocity vector, **T** is the stress tensor, $\mathbf{f}_b$ is the body force.

Constitutive relationship linking the stress tensor deviator $\mathbf{T}^d$ and the rate of deformation $\dot{\mathbf{D}}$ for Bingham fluid is:

$$\dot{\mathbf{D}} = \begin{cases} \dfrac{1}{2\left(\eta + \dfrac{\tau_0}{2\sqrt{II_D}}\right)} \mathbf{T}^d & \text{for } \dfrac{1}{2}\left(\mathbf{T}^d : \mathbf{T}^d\right) > \tau_0^2 \\ 0 & \text{for } \dfrac{1}{2}\left(\mathbf{T}^d : \mathbf{T}^d\right) > \tau_0^2 \end{cases} \tag{3}$$

where τ_0 is the yield stress, η is the plastic viscosity, $II_{\dot{\mathbf{D}}}$ is the second invariant of $\dot{\mathbf{D}}$.

3. SOLUTION TECHNIQUE

The finite volume method is used to solve system of non-linear equations (1) and (2) with constitutive relationship (3). In this section the finite volume discretization of the transport equations is briefly outlined; a more detailed description is available in Demirdžić and Muzaferija [4].

The solution domain is subdivided into a finite number of contiguous control volumes (CV). Numerical evaluation of integrals in equations (1) and (2) requires that coordinates of the cell and face centers, surface vector and cell volume be known. In order to achieve conservative discretization the convection and diffusion fluxes are unique associated with cell faces. The linear spatial variation of dependent variables is assumed in approximating cell-face values and cell-face gradients, and the mid-point rule for calculating integrals is used. An iterative segregated procedure with decoupling is applied. Equations for dependent variables u, v and p are linearised and temporarily decoupled by assuming that coefficient and source terms are known resulting in systems of linear algebraic equations for each dependent variable. These systems are then solved using conjugate gradient method with preconditioning. After every outer iteration effective viscosity μ_{eff}, described below, is calculated for new values of variables. Procedure is repeated until a converged solution is obtained.

In the case of pipe flow of non-Newtonian slurry, it is possible to define an "effective viscosity" μ_{eff}, defined as the ratio of shear stress to average shear rate at the boundary and is given by:

$$\mu_{eff} = \frac{\tau_w}{8\dfrac{v}{d}}, \tag{4}$$

which is derived by neglecting fourth-power term of Buckingham equation [1] and then rearranging it:

$$\frac{\tau_w}{\left(\dfrac{8v}{d}\right)} = \eta\left(1 + \frac{\tau_0 d}{6\eta v}\right) \tag{5}$$

From relations (4) and (5) following equation is obtained:

$$\mu_{eff} = \eta\left[1 + \frac{\tau_0 d}{6\eta v}\right] \tag{6}$$

Finally from equations (6) and (3) the viscosity for Bingham plastics is as follows:

$$\mu_{eff} = \eta\left[1 + \frac{\tau_0}{2\eta\sqrt{II_D}}\right] \tag{7}$$

Reynolds number for Bingham plastics based on effective viscosity is then defined as:

$$\mathrm{Re}_B = \frac{\rho v d}{\mu_{eff}} \tag{8}$$

4. APPLICATION OF THE METHOD

Despite the fact that internal flows of viscoplastic fluids are common in industrial applications, the knowledge base, number and extent of reports are quite limited at the present. Thus, an improved understanding of the pertinent physical and geometric parameters on the flow field can result in better product quality as well as improved process efficiency. The flow of viscoplastic fluids through a circular pipe is frequently encountered in practice affording a geometry that is simple enough to formulate a numerical scheme in the cylindrical coordinate system.

In the present study, internal flow of fly ash slurry at high concentration trough a straight circular pipe of 53 mm inner diameter is analyzed. The flow at the inlet ($x = 0$) is assumed to be fully developed, steady, laminar, incompressible flow of a Bingham fluid in a pipe and is given in its non-dimensional form. At the exit plane of the computational domain flow is assumed to be fully developed. Two-dimensional solution domain is sub-divided into CV's by a uniform numerical mesh. In the Figure 1 a velocity profile post processed in flow2d after numerical calculation is shown. In Figure 2 fully developed velocity profiles at pipe exit for fixed mean flow velocity and various slip velocities at the pipe walls are given as the boundary conditions in the numerical simulation.

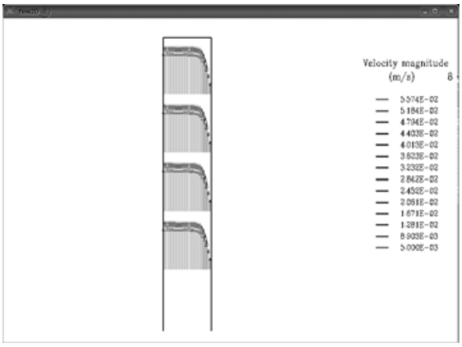

Figure 1. Velocity profile of a fly ash slurry flow post processed in flow2d

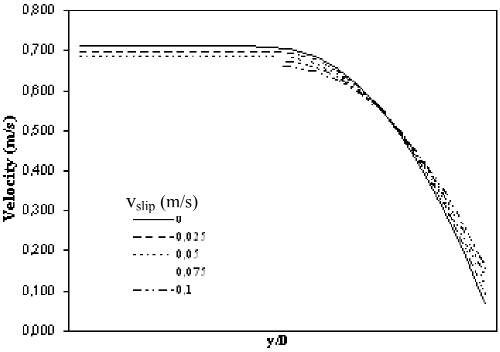

Figure 2. Fully developed velocity profiles for various slip velocities at the pipe wall

The convergence rate of numerical scheme depending on the yield number Y defined by relation (9) is demonstrated in Figure 3 for two values of Reynolds number (10 and 50). From the figure can be seen that more iterations are required for larger yield numbers for the same convergence criterion.

$$Y = \frac{\tau_0 d}{2\eta v} \tag{9}$$

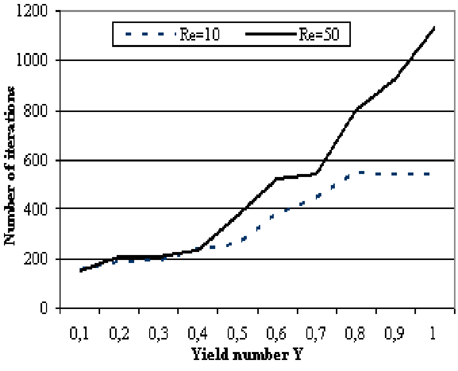

Figure 3. Convergence history for Re=10 and Re=50

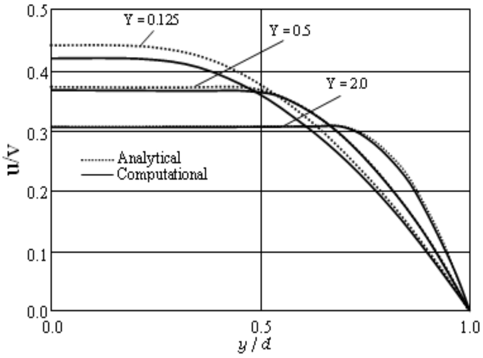

Figure 4. Fully developed velocity profile at pipe exit

Figure 4 shows the fully developed velocity profiles at the exit of the pipe for different yield numbers. The computationally obtained profiles are very close to the exact analytical profiles with the maximum difference less then 2 %. It is interesting to note that the disagreement between the analytical and the computational solutions is larger in the central portion of the pipe and it vanishes near the wall. This is a desirable outcome since, typically, the near wall flow characteristics are the ones of engineering importance.

Rheological properties for the mixture of fly ash and bottom ash slurries with concentrations of 60 and 65 % that are modeled are obtained from the work by Chandel et al. [2]. Figure 5 shows the pressure drop for variable slip velocities at the pipe walls for fly ash slurries at two solid concentrations: 60 and 65 %. It was observed that pressure drop decreases with increasing value of the assumed slip velocity, for the same solid concentration of the mixture. It is also shown that with increased solid concentration the pressure drop is increasing significantly.

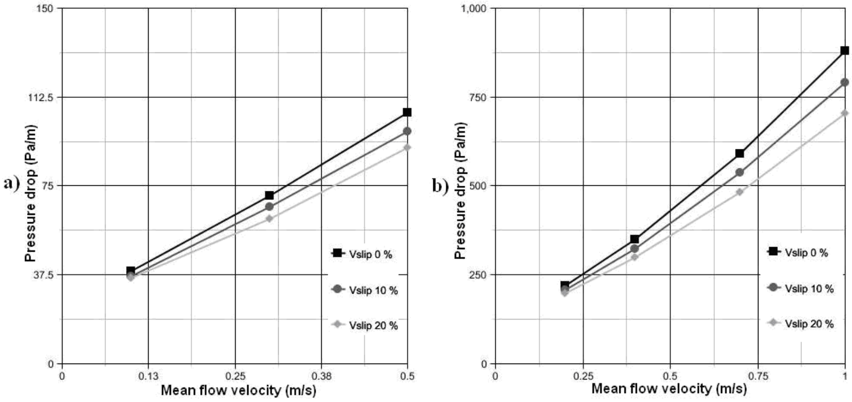

Figure 5. Pressure drop for variable slip velocities at the pipe walls: 0, 10 and 20 % of the mean flow velocity, for fly ash slurries at two solid concentrations a) 60 %, b) 65 %

5. CONCLUSIONS

This study indicates that finite volume method, which is the most widely used method in analysis of Newtonian fluid flows, can be adopted for the solution of complicated highly non-Newtonian flows with success. Numerically obtained results are comparable with experimental results given by Knežević et al. [5]. This work will be continued by modeling the friction on the pipe walls with the aim of obtaining values of slip velocities on the walls simultaneously computed during the simulation.

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