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EFFECT OF TENON DEPTH ON FLEXIBILITY OF MORTISE AND TENON JOINT

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ABSTRACT
This paper presents numerical analysis of the effects of tenon depth on flexibility of mortise and tenon joints. Numerical calculations are carried out with a linear elastic model for orthotropic material. Mathematical model is solved by finite element method. The results of the calculation indicate that a mortise and tenon joint becomes stiffer as tenon depth is increased. A satisfactory agreement was found between the experimental data taken from literature and obtained results, thus confirming the conclusions that were made.
Keywords: wood, mortise and tenon joint, finite element method, stiffness

1. INTRODUCTION
Frame structure represents the most widely used type of furniture constructions. Common construction manufactured by connecting wood elements with shape-adhesive joints. The stiffness and strength of frame structure depend on the properties of the employed joints. In order to be able to carry out analysis and optimization of construction it is necessary to know the joints properties. Attempts are being made to analysis and finding solutions that would improve joint properties [1,2,3].

2. RESEARCH OBJCTIVE AND METHODOLOGY
In the common structural analysis of a furniture frame, joints are modelled considering some idealizations. The joints are assumed to be ideally rigid or pinned. In fact, most joints in real wood structures are more or less flexible or semi-rigid. This study undertook to employ numerical method for the analysis of mortise and tenon joint stiffness. The objective was to determine the effects of tenon depth on flexibility of mortise and tenon T-type joints. Physical model (configuration, measurements and material, except glue type) of joint and loading diagram used in this study are taken from literature [3]. Results of rotational stiffness obtained in this study are compared with experimental values.

2.1. Mathematical model
The equation of momentum balance, expressed in the Cartesian tensor notation [4]

$$\int_S \sigma_{ij} n_j dS + \int_V f_i dV = 0 \qquad \text{... (1)}$$

and of the constitutive relation for the elastic material

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} = \frac{1}{2} C_{ijkl}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right) \qquad \text{... (2)}$$

describe the stress and strain of a loaded solid body in the static equilibrium. In the equations above, x_j are Cartesian spatial coordinates, V is the volume of solution domain bounded by the surface S, σ_{ij} is the stress tensor, n_j is the out warded unit normal to the surface S, f_i the volume force, C_{ijkl} the elastic constant tensor components, ε_{kl} the strain tensor, and u_k the point displacement.
In order to complete the mathematical model, the boundary conditions have to be specified. The surface traction f_{Si} and/or the displacement u_S at the domain boundaries are known, i.e.

$$\sigma_{ij} n_j = f_{Si} \text{ and } u_i = u_S . \qquad ... (3)$$

Governing equations (1) combined with the constitutive relations (2) are solved by the numerical method based on the finite element. Calculations in this study are performed by employing the software package CATIA.

2.2. Physical model

Rectangular mortise and tenon joint was selected for examination. Geometries and measurements of the T-type end to side grain joints are shown in Fig. 1. and Tab. 1. All variables except tenon depth were held constant. Tenon depth varied from 12.7 mm to 76,2 mm.

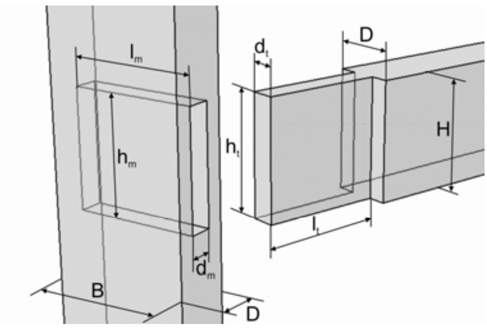

Figure 1. Geometries of the mortise and tenon joint

Table 1. Measurements of the mortise and tenon joint

Post and rail, mm					
B		H		D	
76,2		76,2		25,4	
Tenon, mm			Mortise, mm		
h_t	d_t	l_t	h_m	d_m	l_m
12,7	9,5	25,4	12,9	9,7	l_t+3,87
25,4			25,6		
38,1			38,3		
50,8			51		
63,5			63,7		
76,2			76,4		

Table 2. Elastic properties of maple (Acer saccharum Marsh.)

Modulus of elasticity, GPa			Rigidity modulus, GPa		
E_L	E_R	E_T	G_{LR}	G_{LT}	G_{RT}
13,810	1,311	0,678	1,013	0,753	0,255
Poisson s ratio					
ν_{LR}	ν_{LT}	ν_{RT}	ν_{TR}	ν_{RL}	ν_{TL}
0,46	0,50	0,82	0,42	0,044	0,025

Table 3. Values of the bending moment and shear stress

Tenon depth, mm	M, Nm	τ, MPa
12,7	73,68	0,125
25,4	92,70	0,157
38,1	107,20	0,182
50,8	136,32	0,231
63,5	150,52	0,255
76,2	175,07	0,297

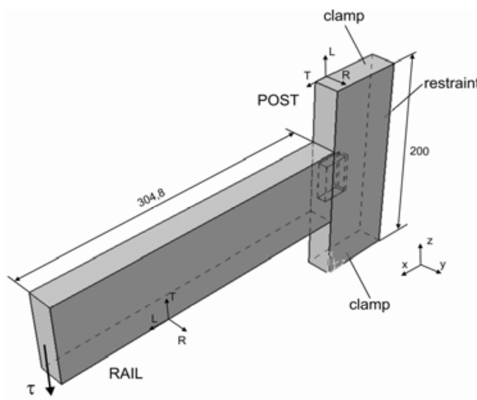

Figure 2. Loading diagram of the mortise and tenon joint

A gap 0,1 mm tick was placed between the teonon and the mortise in which a glue bond was formed. The shoulder of the tenon (rail) was contact connected to the wall of the mortise (post). Calculation was carried out for maple wood (*Acer saccharum Marsh.*). Its elastic properties for wood density ρ=0,57 g/cm^3 and moisture content of 12% are presented in Tab. 2 [5]. Another material component is polyvinyl acetate (PVAC) glue. Selected elastic properties of the glue are E=465,74 MPa and ν=0,29 [2]. Loading diagram of the mortise and tenon joints is shown in Fig. 2. The post of joint was fixed at upper and lower end. Translation of the post lateral surface points was fixed in restrained direction *x*. Loaded end of the rail was at a distance of 304,8 mm from the face of the post. Appropriate value (1/3 of ultimate load [3]) of bending moment was transformed into shear stress on rail cross section surface. Values of the bending moment acting on the joint and appropriate shear stress are presented in Tab. 3.

3. RESULTS

The results of the numerical calculation revealed that the tenon underwent rotation and deflection. Points at the upper tenon surface slipped outside the mortise and are moved downwards, while points at the bottom tenon surface slipped inside the mortise and moved downwards, Fig 3. The effect of the interaction between the rail and post is results both from the bending of the tenon and torsion of the glue line.

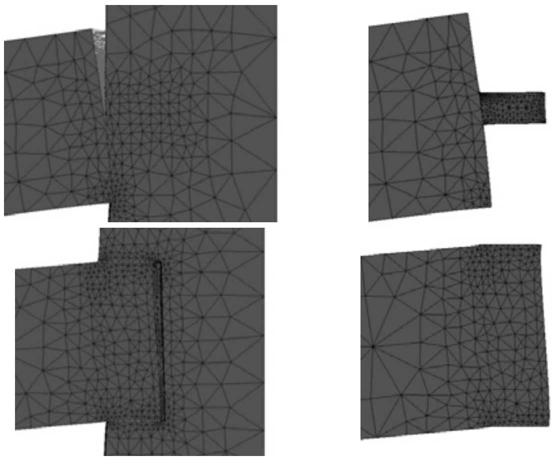

Figure 3. Joint deformations - tenon depth: 12,7 mm (up), 76,2 mm (down; symmetry plane); scale 20:1

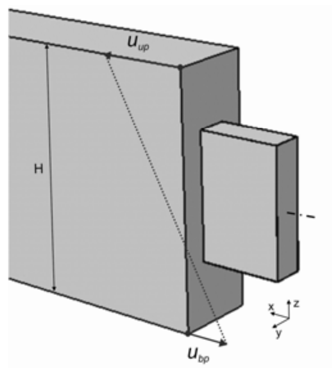

Figure 4. Position of the selected points

The effect of the point position in the upper and bottom edge shoulder of tenon on the change in the point displacement was assessed. Position of the selected points is shown in Fig. 4. The rotation that occurs in semi-rigid joints is described by rotational stiffness R. Angle change of selected joints was evaluated through the use of displacement results and rail width, Fig.4. Values of the rotational stiffness were determined by means of the expression:

$$R = \frac{M}{\phi} = \frac{H}{u_{up} + u_{bp}} \cdot M \qquad \text{... (4)}$$

where: M - the bending moment acting on the joint (Nm), ϕ - the angle change resulting from the joint behaviour, u_{up} and u_{bp} - the absolute values of the upper and bottom point *u* displacement (mm) and H - the rail with (mm).

The effect of the tenon depth on the change in rotational stiffness R was assessed. Values of rotational stiffness R for the six tenon depth are given in Tab. 4 and shown in Fig. 5. Description of test and detailed experimental results can be found in [3].

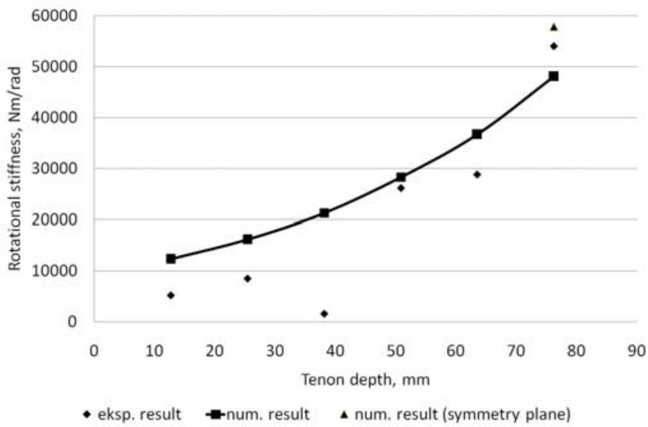

Figure 5. Rotational stiffness R for the six tenon depth

These results indicate that the joint become increasingly stiffer as tenon depth is increased. For tenon depth 12,7 mm, the joint reached 9,7% (according to experimental results), 21,2% (symmetry plane) and 25,5% (outer plane) of the rotational stiffness it would have if the tenon depth was 76,2 mm. Differences between rotational stiffness obtained experimentally and numerical calculation are wide ranged.

Table 4. Rotational stiffness R for the six tenon depth

Rotational stiffness, Nm/rad				
Tenon depth, mm	Exp. results*	Numerical results		Differ., %
12,7	5250,93		12310,70	134,45
25,4	8547,66		16193,77	89,45
38,1	1587,52**		21311,57	
50,8	26151,35		28347,76	8,40
63,5	28826,53		36701,97	27,32
76,2	54066,99	simmetry plane	57916,09	7,12
		outer plane	48113,15	-11,01
* Experimental results are taken from [3]; ** Fault				

4. CONCLUSION

The performed investigations revealed that the numerical procedure used in the study provide a convenient method of obtaining the information needed for determining rotational stiffness for shape-adhesive wood joints. The research model could be used for optimization of joints in wood constructions. Many variables affecting the stiffness of joints could be evaluated individually or it could be analysed the interaction of variables on the joint properties. The knowledge of joint properties brings product engineers to simulate real behaviour wood structure under service loads.

On the basis of the performed numerical calculation, the following conclusions were drawn. It is evident from the analysis of rotational stiffness that the joints become stiffer as tenon depth is increased. Similarity between experimental and numerical results of rotational stiffness allows conclusion that the research model was designed correctly.

5. REFERENCES

[1] Prekrat, S., Španić, N.: Scientific methods for determination of wooden corner joint designs, Drvna industrija, Vol. 60, No. 4, pp. 245-251, 2009.,

[2] Hajdarević, S., Šorn, Š.: Effect of the Spread Adhesive Thickness and Surface on Strength and Stiffness of Joint, Proceedings 16th International Research/Expert Conference „Trends in the Development of Machinery and Associated Technology“, 10-12 September 2012., Dubai, UAE

[3] Erdil, Y.Z., Kasal, A., Eckelman, C.A.: Bending moment capacity of rectangular mortise and tenon furniture joints, Forest Products Journal, Vol. 55, No. 12, pp. 209-213, 2005.,

[4] Chung, T. J.: Applied Continuum Mechanics, Cambridge University Press, Cambridge, 1996.,

[5] Boding, J., Jayne, B. A.: Mechanics of Wood and Wood Composites, Krieger publishing Company, Malabar, Florida, 1993.