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DYNAMIC ANALYSIS OF AN OMNI-DIRECTIONAL MOBILE ROBOT

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ABSTRACT
This paper deals with dynamic analysis of mobile robot equipped with three omni-directional wheels. The mobile robots contain subsystems from different physical domains and in the paper bond graphs are chosen as an appropriate technique for modelling and simulation. Model of Robotino mobile robot is developed and simulation is performed using the program BondSim, developed by the first author of the paper. Program supports systematically model development, based on component model approach, and automatically generation of mathematical model in the form of differential algebraic equations (DAEs) using symbolic manipulation routines.
Keywords: mobile robot, omni-direction wheel, mechatronics design, bond graphs

1. INTRODUCTION
One of the problems of modern metal industry is the existence of a large gap between the highly automated production realized by CNC machine tools, machining centres on one side and non-automated transport between these so-called 'islands of automation' from the other side. The use of an automatic vehicle could be the right solution to solve the problem. Automated vehicles are usually dedicated microprocessor systems - mobile robots. The mobile robots are also increasingly used as entertainment objects and for service and inspection operations.
Taking into account their purpose, one of main performances of mobile robot is its capability of tracking or finding the right ways to avoid obstacles. Researches in field of mobile robotics are increasingly gaining in importance [3-4,6-8]. Control of omnidirectional mobile robots is subject of research in [3,4,9]. Dinh at al propose a tracking control method for three-wheeled omni-directional mobile robot using differential sliding mode controller in [4] and with disturbance and friction in [3], where mobile robot consists of the platform and manipulator (SCARA). A method based on Genetic-Algorithms is proposed to design a fuzzy control of mobile robot in [9]. In [7] authors develop models of mobile robots with three and four wheels. In [9] it has been presented a trajectory generation algorithm for omni-directional vehicles. Mobile robot *Robotino* is analyzed in [6], the objective is to design full-order Luenberger observer by bond graphs, based on causalities and an attention is reserved to the kinematic consideration. In this paper is also used bond graph technique, but without causalities [1]. In the second part of this paper dynamic model of mobile robot is derived, its bond graph model is develop in the third section. The fourth section deals with numerical validation of proposed model.

2. MODEL OF OMNI-DIRECTIONAL MOBILE ROBOT
Omni-directional mobile robot Robotino (Fig.1) consists of laser welded stainless steel platform - chassis equipped with three omni-directional wheels that provide very good manoeuvrability and enhanced mobility compared with traditional wheels.

Figure 1. Mobile robot Robotino

Wheels are located at an angle of 120^0 to each other, as shown in Fig. 2a. Each of them is powered by DC motor (denoted by M1, M2 and M3 in Fig.2b) via planetary gear unit with a gear ratio of 16:1 and the toothed belt. An actual value of motor speed, measured by incremental encoder, is compared with desired value and controlled by a PID controller. Two rechargeable 12 V batteries and the command bridge are mounted to the chassis. The command bridge consists of the controller, the I/O module and interface. It is connected with plug connector with other system modules. Robot is equipped with many different sensors as infrared sensors, anti-collision sensor, an analogue inductive proximity sensor, diffuse sensors and camera system. Also, functionality of whole system can be improved by adding new sensors via I/O module [5].

2.1. Dynamic model of DC motors

The dynamic model of DC motor is represented by two equations that describe processes at electrical and mechanical side. The electrical process of a i^{th} permanent DC motor (i=1,2,3) can be described as:

$$e_{ai} = L_a \frac{di_{ai}}{dt} + R_a i_{ai} + e_{emfi}, \ (i=1,2,3) \qquad \text{... (1)}$$

where e_{ai} is the armature voltage, i_{ai} is the armature current, R_a is the armature resistance, L_a is self-inductance and e_{emfi} is the back emf constant of each motor.

The torque acting on the rotor T_{mi} of i^{th} motor is defined in terms of the rotor angular velocity ω_{mi}, the friction coefficient B_m, the rotor mass moment of inertia J_m and the external (load) torque T_{Li} by:

$$T_{mi} = J_m \dot{\omega}_{mi} + B_m \omega_{mi} + T_{Li}, \ (i=1,2,3). \qquad \text{... (2)}$$

The electromechanical conversion can be written as:

$$T_{mi} = k_t \cdot i_{ai}, \quad e_{emfi} = k_t \cdot \omega_{mi} \qquad (i=1,2,3), \qquad \text{... (3)}$$

where k_t is the torque constant. Two power variables, the external torque acting on the shaft from the motor T_{Li} and the rotor angular velocity ω_{mi}, are transformed by the gear box as:

$$T_i = k \cdot T_{Li}, \ \omega_i = (1/k) \cdot \omega_{mi} \ (i=1,2,3), \qquad \text{... (4)}$$

where k is the reduction ratio of gear box.

2.2. Model of robot's wheel

The dynamic model of the i^{th} wheel (i=1,2,3) (Fig.2c) in acceleration phase can be described by:

$$J_w \dot{\omega}_i + R \cdot (F_i + F_{bi}) = T_i, \ (i=1,2,3), \qquad \text{... (5)}$$

where J_w, ω_i, R, F_i, and F_{bi} are the moment of inertia, angular velocity, radius, traction force, and viscous friction force for each wheel, respectively.

Linear velocities of wheels (i=1,2,3) are defined by:

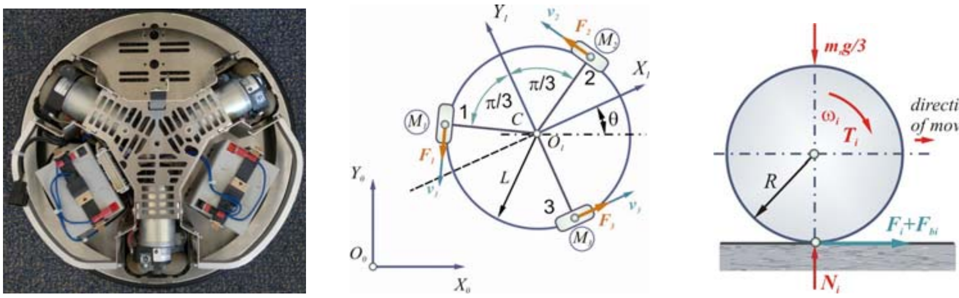
$$v_i = R \cdot \omega_i \ (i=1,2,3). \qquad \text{... (6)}$$

2.3. Model of mobile platform

Configuration of the chassis with powered wheels is depicted in Fig.2b. Three wheels have the same radius R. The distance between the wheel's centre and centre of the platform is denoted by L. The local coordinate frame $O_1X_1Y_1Z_1$ is attached to the platform and its orientation with respect to the global frame $O_0X_0Y_0Z_0$ is defined by rotation matrix $\mathbf{R}_1^0$:

$$\mathbf{R}_1^0 = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}, \ (i=1,2,3). \qquad \text{... (7)}$$

Vectors of wheel linear velocities $\mathbf{v}_i$ (i=1,2,3) can be expressed in the local frame by:



a) b) c)

Figure 2. a) The platform, b) The schematic of the platform, c) The wheel

$$\mathbf{v}_1^1 = \begin{bmatrix} -v_1 \cdot cos(\pi/3) \\ -v_1 \cdot sin(\pi/3) \end{bmatrix}, \mathbf{v}_2^1 = \begin{bmatrix} -v_2 \cdot cos(\pi/3) \\ v_2 \cdot sin(\pi/3) \end{bmatrix}, \mathbf{v}_3^1 = \begin{bmatrix} v_3 \\ 0 \end{bmatrix}, \quad \text{... (8)}$$

and in the global frame (superscript 0 is omitted for all vectors represented in the global frame) as:

$$\mathbf{v}_1 = \begin{bmatrix} -v_1 \cdot cos(\theta + \pi/3) \\ -v_1 \cdot sin(\theta + \pi/3) \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -v_2 \cdot cos(\theta - \pi/3) \\ -v_2 \cdot sin(\theta - \pi/3) \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} v_3\, cos(\theta) \\ v_3\, sin(\theta) \end{bmatrix}. \quad \text{... (9)}$$

Similarly, the traction forces on the wheels $\mathbf{F}_i$ (i=1,2,3) in the global frame can be written as:

$$\mathbf{F}_1 = \begin{bmatrix} -F_1 \cdot cos(\theta + \pi/3) \\ -F_1 \cdot sin(\theta + \pi/3) \end{bmatrix}, \mathbf{F}_2 = \begin{bmatrix} -F_2 \cdot cos(\theta - \pi/3) \\ -F_2 \cdot sin(\theta - \pi/3) \end{bmatrix}, \mathbf{F}_3 = \begin{bmatrix} F_3\, cos(\theta) \\ F_3\, sin(\theta) \end{bmatrix}. \quad \text{... (10)}$$

According to the classical approach of engineering mechanics, the force that wheel exerts on the platform can be represented by a resultant force vector at the mass centre and a resultant moment about it. The translational dynamics governed by the platform mass centre is given by:

$$m_r \frac{d\mathbf{v}_c}{dt} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3, \quad \text{... (11)}$$

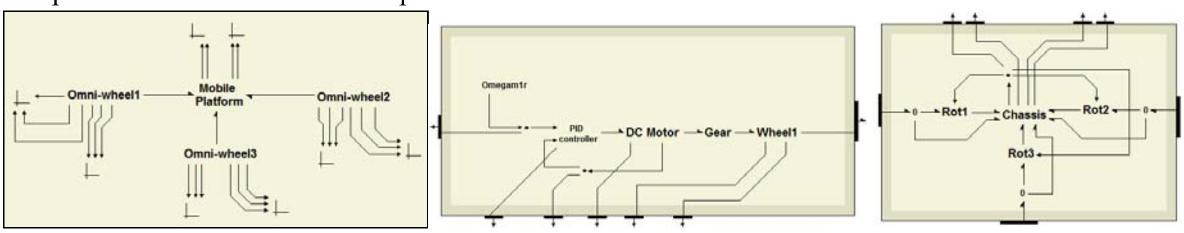
where m_r is the robot mass and $\mathbf{v}_c$ is velocity of the mass centre. Rotational dynamics is described by:

$$J_r \frac{d\omega}{dt} = (F_1 + F_2 + F_3) \cdot L, \quad \text{... (12)}$$

where J_r and ω are the centroidal mass moment of inertia and angular velocity of the robot.

3. BOND GRAPH MODEL OF THE MOBILE ROBOT

Bond graph model of omni-directional mobile robot is depicted in Fig.3a. Model of powered omni-directional wheel on the next level of decomposition, as shown in Fig.3b, consists of models of PID controller, DC motors, gear box and wheel. The electromechanical conversion of DC motor is realized by the gyrator **GY** bond graph component, described by Eq.(3). Fig.3c shows structure of component **Mobile platform**. Three components **Roti** (i=1,2,3) form traction velocities and forces using modulated transformer components **MTR**, as defined by Eqs (9) and (10). The translational and rotational dynamics, according to Eqs. (11) and (12), is presented by the inertia bond graph components in framework of component **Chassis**.

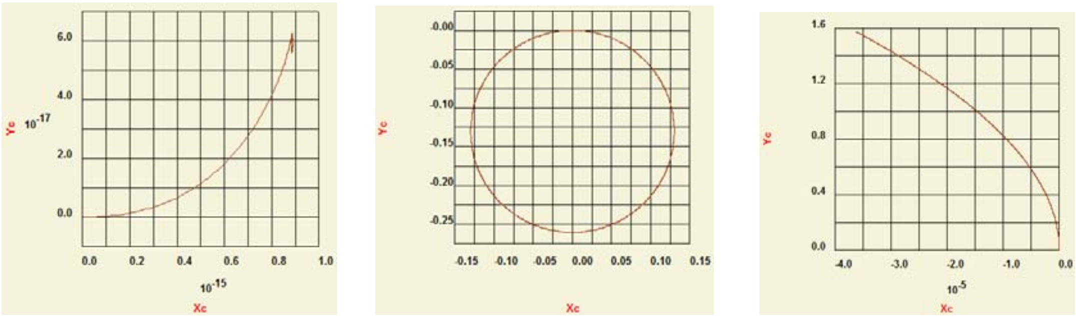


a) b) c)

Figure 3. Bond graph model: a) Mobile robot, b) Omni-wheel1, c) Mobile Platform

4. SIMULATION RESULTS

To verify developed model the simple numerical experiment is performed. If all motors are actuated the same value of rotational velocity robot only rotates about axis trough the mass centre (Fig.4a). If motor M3 is not actuated and motors M1 and M2 are actuated the same value of rotational velocity the robot follows circle trajectory (Fig.4b). Line forward motion of robot can be obtained if motors M1 and M2 are actuated the same value of rotational velocity, but the motor M1 with negative value (Fig.4c) [5]. Simulation period is taken of 3.5 s with the time step of 1ms. (Parameters of models are: L_a=0.004 H, R_a=2 Ω, J_m=71e-7 kgm2, B_m=1e-5 Nms, m_r=11 kg, R=0.04m and L=0.13 m). Mathematical model is generated in form of differential algebraic equation (DAE) and solved by backward differentiation formulae (BDF).



a) b) c)

Figure 4. Simulation results for: a) $\omega_{mi}=50\pi$ rad/s (i=1,2,3); b) $\omega_{mi}=50\pi$ rad/s (i=1,2) $\omega_{m3}=0$ rad/s; c) $\omega_{m1}=-50\pi$ rad/s $\omega_{m2}=50\pi$ rad/s, $\omega_{m3}=0$ rad/s

5. CONCLUSION

Bond graph model of omni-directional mobile robot is developed in this paper. Proposed algorithm should be also used for modelling and simulation of any other mechatronics system. Applied bond graph technique provides modelling of systems composed of subsystems from different physical domains in an unique way describing the power flow through the system. Model is developed and simulation is carried out using the program BondSim [1]. Different to other authors, the causalities, well known in bond graph theory, are not taken into consideration explicitly, which enables relatively simple treatment of very complex systems. The simulation results presented illustrates the applicability of the developed model of omni-directional mobile robot. The developed model of omni-directional mobile robot can served for future investigation in field of mobile robotics – analysis of different control strategies, verification of the navigation performances of mobile robot, etc.

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