ASSESSMENT OF FATIGUE CRACK GROWTH IN PIPES SUBJECTED TO VARIABLE LOADING

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ABSTRACT
Computer simulation of fatigue crack growth in pipes subjected to variable amplitude loading is considered in this work. The fatigue crack, detected during the inspection, significantly reduces remaining fatigue life. The fatigue life is calculated by solving the equation of fatigue crack growth rate step-by-step from initial to final crack size by Runge-Kutta method. The computer program, based on this procedure, is used for the fatigue crack growth simulation.

Keywords: variable amplitude loading, steel pipes, crack-like defect, assessment of remaining fatigue life

1. INTRODUCTION
Fatigue is one of the most frequent form of the failures of the structural details, machine elements, pressure vessels and piping systems. According to ref. [1,2,3] 50-90 percent of all mechanical failures are fatigue failures. This study focuses on the assessment of fatigue crack growth in pipes subjected to variable loading. The fracture mechanics approach was utilized, average material properties were assumed.

2. ANALYSIS OF CRACK PROPAGATION
2.1. Crack propagation model
The crack propagation lives were calculated with the Paris equation [4]:

\[
\frac{da}{dN} = C(\Delta K)^m
\] (1)

where

\begin{align*}
\frac{da}{dN} &= \text{crack growth rate,} \\
\Delta K &= \text{range of stress intensity factor,} \\
C \text{ and } m &= \text{material constants.}
\end{align*}

2.2. Stress intensity factor
The stress intensity factor was calculated by using Raju and Newman solution [5] for internal surface longitudinal cracks in pipes (Fig.1) Eq.(2):

\[
K_I = \frac{pR}{t} \sqrt{\frac{a}{Q}} f_i \left( \frac{a}{c}, \frac{a}{t}, R \phi \right)
\] (2)

where:
Figure 1. Pipe with longitudinal internal surface crack

\[ Q = 1 + 1.464 \left( \frac{a}{c} \right)^{1.65} \]  \hspace{1cm} (3)

\[ F_i = \frac{t}{R} \left( \frac{R_0^2}{R^2} - \frac{1}{2 R} \right) \left[ 2 G_0 - 2 \left( \frac{a}{R} \right) G_1 + 3 \left( \frac{a}{R} \right)^2 G_2 - 4 \left( \frac{a}{R} \right)^3 G_3 \right] \]  \hspace{1cm} (4)

where \( a \) = depth of surface crack, \( 2c \) = surface crack length, \( t \) = cylinder wall thickness, the shape factor for an elliptical crack, \( Q \) is the square of the complete elliptic integral of the second kind and is approximated by Eq.(3), \( p \) = internal pressure in cylinder, \( R, R_0 \) = inner and outer radii of cylinder. Influence coefficient for \( j \)-th stress distribution on crack surface, \( G_j \), was obtained from the appropriate finite element solution and given in tables [5] for the particular values of \( t/R, a/c, a/t \). \( G_j \) values for another \( a/t \) values was determined in this work by using regression analysis:

\[ G_0 = 0.90933 + 0.6433 \left( \frac{a}{t} \right) \]

\[ G_1 = 0.60133 + 0.27 \left( \frac{a}{t} \right) \]

\[ G_2 = 0.48133 + 0.1566 \left( \frac{a}{t} \right) \]

\[ G_3 = 0.409 + 0.1166 \left( \frac{a}{t} \right) \]  \hspace{1cm} (5)

3. PREDICTION OF THE REMAINING FATIGUE LIFE

Average material properties, for steel, were assumed: \( m = 3, C = 4.9 \cdot 10^{-12} \), with \( \Delta K \) in units of MPa \( \sqrt{m} \) and \( da/dN \) in units of m/cycle, threshold stress intensity factor \( \Delta K_{th} = 4 \) MPa \( \sqrt{m} \), the fracture toughness \( K_c = 55 \) MPa \( \sqrt{m} \). Geometrical parameters are: \( t/R = 0.25, t = 10 \text{ mm}, R = 40 \text{ mm}, R_0 = 50 \text{ mm}, a/c = 0.4 \), initial crack size \( a_0 = 0.4 \) mm. The pipes are subjected to variable internal pressure (Table 1 and 2). The remaining fatigue life (crack propagation life) \( N_p \) is obtained by solving Eq.(1) using the Runge-Kutta [6] method. Obtained results are shown by \( \Delta p \) vs N diagram in Fig.2 and compared with Miner's and Haibach's results.

Table 1. Heavy spectrum of internal pressure ranges

<table>
<thead>
<tr>
<th>Block number, ( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency, ( \gamma )</td>
<td>0.003</td>
<td>0.007</td>
<td>0.008</td>
<td>0.014</td>
<td>0.024</td>
<td>0.044</td>
<td>0.078</td>
<td>0.138</td>
<td>0.248</td>
<td>0.436</td>
</tr>
<tr>
<td>Normalized pressure range, ( \Delta p_i / \Delta p )</td>
<td>1</td>
<td>0.944</td>
<td>0.927</td>
<td>0.906</td>
<td>0.850</td>
<td>0.800</td>
<td>0.770</td>
<td>0.735</td>
<td>0.680</td>
<td>0.630</td>
</tr>
</tbody>
</table>
Table 2. Medium spectrum of internal pressure ranges

<table>
<thead>
<tr>
<th>Block number, ( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
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<th>10</th>
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<td>0.044</td>
<td>0.078</td>
<td>0.138</td>
<td>0.248</td>
<td>0.436</td>
</tr>
<tr>
<td>Normalized pressure range, ( \Delta p_i / \Delta p_1 )</td>
<td>1</td>
<td>0.890</td>
<td>0.852</td>
<td>0.814</td>
<td>0.767</td>
<td>0.690</td>
<td>0.600</td>
<td>0.510</td>
<td>0.420</td>
<td>0.340</td>
</tr>
</tbody>
</table>

Figure 2. Comparison of predicted fatigue lives with Miner's and Haibach's results for heavy spectrum

Table 3. Predicted fatigue lives for variable-amplitude fatigue \( N_{pa} \) cycles

<table>
<thead>
<tr>
<th>Equivalent pressure range, ( \Delta p_{RMC} ), MPa</th>
<th>Heavy spectrum</th>
<th>Medium spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miner rule or Haibach’s rule</td>
<td>Computer simulation</td>
<td>Difference, %</td>
</tr>
<tr>
<td>70.0</td>
<td>76467</td>
<td>70003</td>
</tr>
<tr>
<td>60.0</td>
<td>121507</td>
<td>115003</td>
</tr>
<tr>
<td>45.0</td>
<td>288372</td>
<td>279003</td>
</tr>
<tr>
<td>35.0</td>
<td>613559</td>
<td>618003</td>
</tr>
<tr>
<td>26.0</td>
<td>1637898</td>
<td>1600003</td>
</tr>
<tr>
<td>22.0</td>
<td>3381058</td>
<td>3850316</td>
</tr>
<tr>
<td>20.0</td>
<td>5818038</td>
<td>8930003</td>
</tr>
<tr>
<td>18.5</td>
<td>7793835</td>
<td>23945003</td>
</tr>
<tr>
<td>16.0</td>
<td>11296608</td>
<td>12239100</td>
</tr>
<tr>
<td>14.0</td>
<td>21010953</td>
<td>30700003</td>
</tr>
</tbody>
</table>

The following formulas were used: Miner’s [7], if all pressure ranges are above fatigue limit

\[
N = \frac{N_1}{\sum_{i=1}^{10} \gamma_i \left( \frac{\Delta p_i}{\Delta p_1} \right)^m}
\]

and Haibach’s [8], which accounts for the damaging effects of pressure ranges below the constant amplitude (CA) fatigue limit

\[
N = \frac{N_1}{\sum_{i=1}^{k} \gamma_i \left( \frac{\Delta p_i}{\Delta p_1} \right)^m + \left( \frac{\Delta p_1}{\Delta p_{CAFL}} \right)^m - \sum_{i=k+1}^{n} \gamma_i \left( \frac{\Delta p_i}{\Delta p_1} \right)^{2m-1}}
\]
where equivalent pressure range is

\[
\Delta p_{RMC} = \left[ \sum_{i=1}^{n} \gamma_i \left( \frac{\Delta p_i}{\Delta p_1} \right)^m \right]^{1/m} \Delta p_1
\]

(8)

The similar procedure was performed for medium spectrum:

![Diagram](image-url)

4. CONCLUSIONS

When all pressure ranges are above CA fatigue limit results of computerized simulation are identical to those obtained by Miner rule (difference is less than 1%) as derived analytically by Maddox [9]. This is valid only if interval \(a_i\) to \(a_f\) is the same for VA and CA load. If the final crack size is determined by \(K_c\) than Miner’s results become non-conservative: 9% for heavy spectrum and 30% for medium spectrum. When some of the pressure ranges are below CA fatigue limit Haibach’s results are conservative for heavy spectrum and non-conservative for medium spectrum.

5. REFERENCES

[8] Haibach E.: Modifizierte lineare schadensakkumulations-hypothese zur berucksichtigung des dauerfestigkeitsabfalls mit fortschreitender schadigung, Laboratorium fur betriebsfestigkeit, TM. No.50/70, Darmstadt, Germany, 1970