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MATHEMATICAL MODELING OF SURFACE COEFFICIENT OF HEAT TRANSMISSION BY CYLINDRICAL AND SPHERICAL WALL

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ABSTRACT
The aim of this paper is the computation of the amount of heat that transmitted through a) cylindrical and b) spherical wall, in function of the inside and outside surface of the wall. Through mathematical modeling we see that the overall coefficient of heat transmission differs when analyzed for: 1) inside surface and 2) the outside surface of the wall. Modeling and determination of this coefficient in function of wall surfaces is very important in practice in the field of heat transmission of walls i.e. heating, ventilation and air conditioning in buildings. Modeling feature of coefficient of heat transmission in unit W/(m2K) and the amount of heat exchange consists in that because this coefficient in literature is found only in function of the depth of the wall. However in this paper will be compared and analyzed the modeled coefficients by those in thermo-technical literature. These analyses - results will be simulated through appropriate software program and illustrated by diagrams.
Keywords: Coefficient, transmission, heat, cylindrical and spherical wall.

1. INTRODUCTION
In the following we will analyze the overall coefficient of heat transmission and thermal flux to both cylindrical and spherical walls with homogeneous structure in stationary heat conduction through the wall. It is assumed that environments air temperatures (on the inside and on the outside of the wall) which restrict wall are not the same.

2. MODELING THE OVERALL COEFFICIENT OF HEAT TRANSMISSION AND THE HEAT FLOW THROUGH THE CYLINDRICAL AND CONICAL WALLS
Initially we start from the equations of heat convection to both sides of the wall [1]:

$$\left.\frac{\dot{Q}}{\alpha_b \pi d_1^2} = \left(t_b - t_1\right); \quad \frac{\dot{Q}}{\alpha_j \pi d_2^2} = \left(t_2 - t_j\right)\right\} \qquad \dots (1)$$

Also we know that the differential equation of the temperature field is [2]:

$$\frac{dt}{d\tau} = a\nabla^2 t\left(\pm\frac{\dot{q}_b}{c_p \rho}\right) = a\Delta t\left(\pm\frac{\dot{q}_b}{c_p \rho}\right) \qquad \dots (2)$$

2.1. Cylindrical wall
The expression ∇^2t in cylindrical coordinates is:

$$\nabla^2 t = \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2} + \frac{\partial^2 t}{\partial z^2} \qquad \dots (3)$$

Where: r – radius vector; φ – polar angle, and z – applicate.

- When the temperature does not depend on time - stationary conduction, does not depend on the polar angle φ and z coordinates, temperature field differential equation (2) takes the form:

$$\frac{d^2t}{dr^2}+\frac{1}{r}\frac{dt}{dr}=0 \qquad \dots (4)$$

After integration of equation (4), for certain boundary conditions, achieved expression:

$$\frac{\dot{Q}}{2\pi\lambda l}\ln\frac{d_2}{d_1}=\left(t_1-t_2\right) \qquad \dots (5)$$

By collection of equations (1) and (5) we achieved the thermal flux:

$$\dot{Q}=\frac{\pi\cdot l\left(t_b-t_j\right)}{\frac{1}{\alpha_b d_1}+\frac{1}{2\lambda}\ln\frac{d_2}{d_1}+\frac{1}{\alpha_j d_2}} \qquad \dots (6)$$

In general literature the thermal flux has the form [3]:

$$\dot{Q}=\frac{l\cdot\left(t_b-t_j\right)}{\frac{1}{d_1\pi\alpha_b}+\frac{1}{2\pi\lambda}\ln\frac{d_2}{d_1}+\frac{1}{\alpha_j\pi d_2}}=l\cdot k\left(t_b-t_j\right) \qquad \dots (7)$$

Where:

$$k=\frac{1}{\frac{1}{d_1\pi\alpha_b}+\frac{1}{2\pi\lambda}\ln\frac{d_2}{d_1}+\frac{1}{d_2\pi\alpha_j}}\ ,\ \frac{W}{mK} \qquad \dots (8)$$

If equation (6) multiplied by the d_1/d_1 will be achieved the thermal flow rate in function of inside surface area and overall heat transfer coefficient:

$$\dot{Q}=\frac{\pi\cdot l\cdot d_1\left(t_b-t_j\right)}{\frac{1}{\alpha_b}+\frac{d_1}{2\lambda}\ln\frac{d_2}{d_1}+\frac{1}{\alpha_j}\frac{d_1}{d_2}}=F_b k_b\left(t_b-t_j\right) \qquad \dots (9)$$

Or if equation (6) multiplied by the d_2/d_2 will be achieved the thermal flow rate in function of outside surface area and overall heat transfer coefficient:

$$\dot{Q}=\frac{\pi\cdot l\cdot d_2\left(t_b-t_j\right)}{\frac{1}{\alpha_b}\frac{d_2}{d_1}+\frac{d_2}{2\lambda}\ln\frac{d_2}{d_1}+\frac{1}{\alpha_j}}=F_j k_j\left(t_b-t_j\right) \qquad \dots (10)$$

Where: k_b, k_j, W/(m^{2}K) – overall heat transmission coefficients (inside and outside) to the cylindrical wall:

$$k_b=\frac{1}{\frac{1}{\alpha_b}+\frac{d_1}{2\lambda}\ln\frac{d_2}{d_1}+\frac{1}{\alpha_j}\frac{d_1}{d_2}}\ ;\ k_j=\frac{1}{\frac{1}{\alpha_b}\frac{d_2}{d_1}+\frac{d_2}{2\lambda}\ln\frac{d_2}{d_1}+\frac{1}{\alpha_j}} \qquad \dots (11)$$

Where:

- $F_b=\pi d_1 l$, m^2 – inside surface area of a cylinder;
- $F_j=\pi d_{n+1} l$, m^2 – outside surface area of a cylinder

2.2. Spherical Wall

The expression $\nabla^2 t$ in spherical coordinates is:

$$\nabla^2 t=\frac{\partial^2 t}{\partial r^2}+\frac{2}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial\psi^2}+\frac{\cos\psi}{r^2\sin\psi}\frac{\partial t}{\partial\psi}+\frac{1}{r^2\sin\psi}\frac{\partial^2 t}{\partial\psi^2} \qquad \dots (12)$$

Ku: r – radius vector; φ – polar angle, dhe ψ – angle geographical length.

- When the temperature does not depend on time - stationary conduction, and does not depend on the polar angles φ and ψ, temperature field differential equation (2) takes the form

$$\frac{d^2t}{dr^2}+\frac{2}{r}\frac{dt}{dr}=0 \qquad \dots (13)$$

After integration of equation (b.0), for certain boundary conditions, achieved the expression:

$$\frac{\dot{Q}}{2\pi\lambda}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)=\left(t_1-t_2\right) \qquad \dots (14)$$

By collection of equations (1) and (14) we achieved the thermal flux:

$$\dot{Q}=\frac{\pi\cdot\left(t_b-t_j\right)}{\frac{1}{\alpha_b d_1^2}+\frac{1}{2\lambda}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)+\frac{1}{\alpha_j d_2^2}} \qquad \dots (15)$$

In general literature the thermal flux has the form:

$$\dot{Q}=\frac{\left(t_b-t_j\right)}{\frac{1}{\alpha_b\pi d_1^2}+\frac{1}{2\pi\lambda}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)+\frac{1}{\alpha_j\pi d_2^2}}=k\left(t_b-t_j\right) \qquad \dots (16)$$

Where:

$$k=\frac{1}{\frac{1}{\alpha_b\pi d_1^2}+\frac{1}{2\pi\lambda}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)+\frac{1}{\alpha_j\pi d_2^2}}\ ,\ \frac{W}{K} \qquad \dots (17)$$

If equation (15) multiplied by the d_1^2/d_1^2 will be achieved the thermal flow rate in function of inside surface area and overall heat transfer coefficient:

$$\dot{Q}=\frac{\pi\cdot d_1^2\left(t_{bp}-t_{jp}\right)}{\frac{1}{\alpha_b}+\frac{d_1^2}{2\lambda}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)+\frac{1}{\alpha_j}\left(\frac{d_1}{d_2}\right)^2}=F_b k_b\left(t_b-t_j\right) \qquad \dots (18)$$

Or if equation (15) multiplied by the d_2^2/d_2^2 will be achieved the thermal flow rate in function of outside surface area and overall heat transfer coefficient:

$$\dot{Q}=\frac{\pi\cdot d_2^2\left(t_{bp}-t_{jp}\right)}{\frac{1}{\alpha_b}\left(\frac{d_2}{d_1}\right)^2+\frac{d_2^2}{2\lambda}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)+\frac{1}{\alpha_j}}=F_j k_2\left(t_b-t_j\right) \qquad \dots (20)$$

Where:

- k_b, k_j, W/(m^{2}K) - overall heat transmission coefficients (inside and outside) to the spherical wall:

$$k_b=\frac{1}{\frac{1}{\alpha_b}+\frac{d_1^2}{2\lambda}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)+\frac{1}{\alpha_j}\left(\frac{d_1}{d_2}\right)^2}\ ;\ k_j=\frac{1}{\frac{1}{\alpha_b}\left(\frac{d_2}{d_1}\right)^2+\frac{d_2^2}{2\lambda}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)+\frac{1}{\alpha_j}} \qquad \dots (21)$$

$F_b=4\pi r_1^2=\pi d_1^2$, m^2 – inside surface area of a sphere;
$F_j=4\pi r_2^2=\pi d_2^2$, m^2 – outside surface area of a sphere;

Where: $\dot{Q}$, W – the amount of heat that passes through any wall surface-element; t_{bp}, ^{0}C – ambient air temperature of the inside closed environment; t_j, ^{0}C – ambient air temperature of the outside closed environment; α_b, α_j, W/(m^{2}K) – heat convection (inside and outside) coefficients; λ, W/(mK) – coefficient of thermal conductivity through the wall.

3. ANALYSIS OF OVERALL HEAT TRANSMISSION COEFFICIENTS AND THERMAL FLOW

Below are some diagrams for the overall heat transmission coefficients and thermal flow in function of thermal conductivity coefficient, fig.1 to the cylindrical wall and fig. 2 to spherical wall, for these values: λ=0...50 W/(mK); α_b=10W/(m^{2}K); α_j=25W/(m^{2}K); d_1=5m; d_2=5,6m; l=3m; t_b=20^{0}C dhe t_j=10^{0}C.

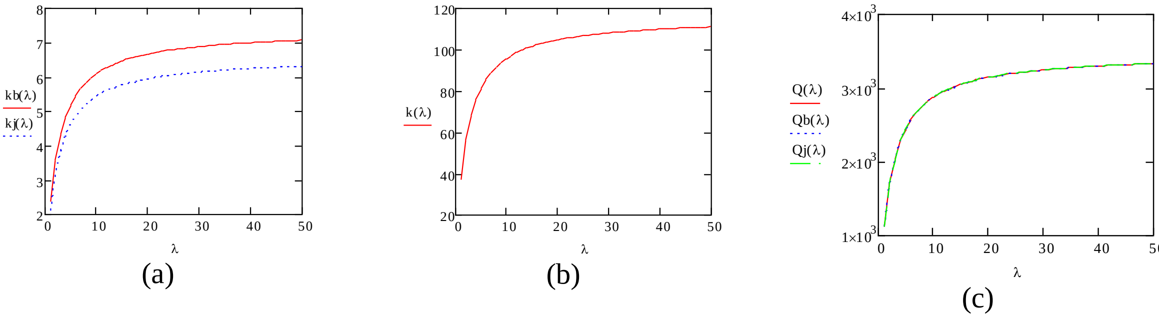

Fig.1. *Overall heat transmission coefficients and thermal flow in function of thermal conductivity coefficient to the cylindrical wall*

Where $F_b=\pi d_1 l$=47,124 m^2 and $F_j=\pi d_2 l$=52,779 m^2

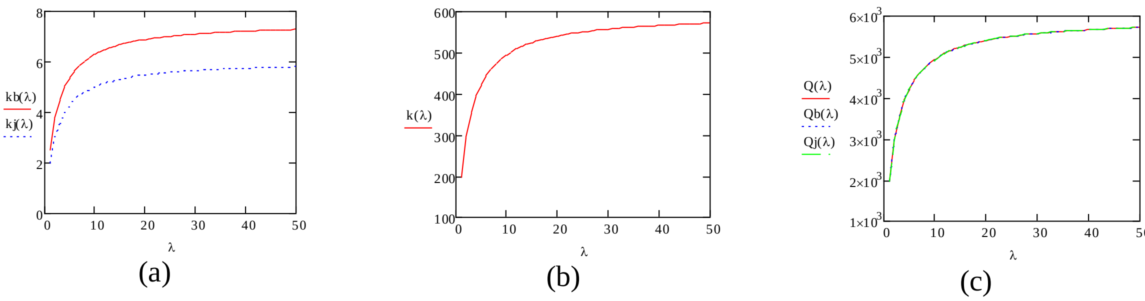

Fig.2. *Overall heat transmission coefficients and thermal flow in function of thermal conductivity coefficient to the spherical wall*

Where $F_b=\pi d_1^2$=78,54 m^2 and $F_j=\pi d_2^2$=98,52 m^2

4. CONCLUSION

Based on the above formulas respectively diagrams, for cylindrical and spherical wall, it seems that the value of transmission coefficient *k*, W/K, taken from the literature is significantly larger than the heat transmission coefficients, k_b, k_j, W/(m^{2}K) – to the inside surface area respectively outside surface area of the wall. While coefficient k_b is greater than k_j, and this means smaller area on the inside wall area F_b versus the outside surface area F_j of the wall. Figures c) argue the validity of using these expressions, while a figures a) and b) the practical importance of their use, since in many cases the analytical and practical thermal treatment of the walls depends on the inside surface or the outside surface area of the wall.

5. REFERENCES

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