

## MODE-MIXITY IN NUMERICAL SIMULATION OF FRMM TEST: LOCAL PARTITIONING USING COHESIVE ZONE

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### ABSTRACT

Fixed-ratio mode-mixity tests were numerically investigated using various cohesive zone properties. The deformation energy (mode I and mode II fracture) going into cohesive elements is monitored and fracture mode-mixities for different configurations are calculated and compared to existing analytical partitioning theories. Opposite to the theories, partitioning is observed to be property dependent.

**Keywords:** mixed-mode fracture, numerical simulation, FRMM test, cohesive zone

### 1. INTRODUCTION

Delamination, as one of the major fracture mechanisms for composite laminates and adhesive joints, is investigated in different fracture tests that make extensive use of beam like geometries. From the experimental results (using appropriate test configuration), the pure mode I and II fracture toughness can be calculated directly; however, analysis and partitioning of test configurations with mixed mode fracture is not at all straightforward. To calculate contributions from mode I and mode II fracture one can implement analytical or numerical methods, each of which suffers from a number of uncertainties and can produce different results depending on choice of a theoretical approach, numerical model etc.

In this work, delamination fracture process in 18 different configurations of fixed-ratio mode-mixity (FRMM) test (Figure 1) are simulated in Abaqus finite element (FE) software package using cohesive zone model. The deformation energy going into cohesive elements (mode I and mode II) is evaluated and fracture mode-mixity for the different configurations is calculated and compared to the existing analytical partitioning theories. The choice of simulation parameters and energy calculation method is based on previous work from authors given in [1].

This work is a part of the Round-robin investigation of wider international activity on mixed-mode fractures in beam-like geometries under the coordination of European Structural Integrity Society, Technical Committee 4 (ESIS TC4). The ultimate goal of this project is a new testing protocol with recommendations for the accurate determination of mode-mixity in all beam-like geometries.

### 2. FRMM TEST, FE MODEL AND SIMULATION CONFIGURATIONS

Figure 1 shows the double cantilever beam (DCB) specimen geometry and FRMM test configuration used in this work. The DCB-FE model is made from two separate beams (parts) with coincident nodes connected along a half of the length (dashed line) with zero-thickness cohesive elements (having nominal thickness equal to 1). Other half of beams have unconnected coincident nodes, representing pre-crack. No surface interaction in pre-crack region is defined since the two pre-crack surfaces are separated immediately at the test initiation. The upper beam height is kept constant and bottom beam height is varied to provide different range of mode

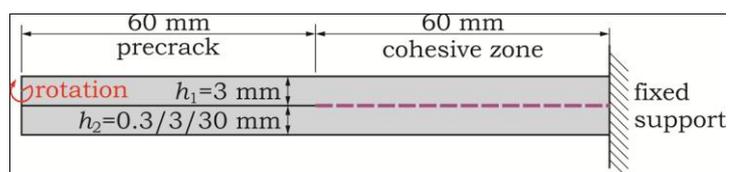


Figure 1. FRMM test configuration.

mixity. The upper beam height is kept constant and bottom beam height is varied to provide different range of mode

mixities. Abaqus CPE4 (4-node bilinear plane strain quadrilateral) elements are used for modelling beams and COH2D4 (4-node two-dimensional cohesive) elements for modelling cohesive zone. The beam material is linear elastic, isotropic with the modulus of elasticity 50GPa and Poisson's ratio 0.38. Rotation is applied incrementally at the end of the top beam which is set to be rigid (nodes at the end line are connected into the rigid body) and the other ends of the beams are fixed. The rotation is chosen because it ensures constant moment loading of upper beam and numerical stability of simulation. It also replicates conditions considered in analytical analyses given in [2] and [3], which are used to compare numerical results.

Cohesive zone response is modelled using a traction-separation model [4], with uncoupled initial linear elastic behaviour defined with elasticity matrix using arbitrarily high set value of stiffness, equal for normal and shear strains (separations);  $K_{nn} = K_{ss} = 10^{15}$  Pa. Element damage initiation is defined using the quadratic nominal stress criterion:

$$\left(t_n / t_n^o\right)^2 + \left(t_s / t_s^o\right)^2 = 1. \quad \dots(1)$$

where  $t_n^o, t_s^o$  represent the peak stress values when the deformation is either purely normal or purely in the shear direction (inter-laminar strength) and equal values for both directions are used  $t_n^o = t_s^o = t^o$ .

The linear damage evolution based on energy is used, with equal critical fracture energy required to cause failure in the pure normal or shear directions and total mixed-mode fracture energy  $G_I^C = G_{II}^C = G^C$ . Dependence of the fracture energy on the mode mixity is defined by the linear law:

$$G_I + G_{II} = G^C, \quad \dots(2)$$

where  $G_I$  and  $G_{II}$  refer to the work done by the traction in the normal and the shear directions, respectively. The choice of equal inter-laminar strength and critical energy for two fracture modes is limitation of the current cohesive model (more details in [2]), but some observations from it can be general.

Eighteen different configurations (table 1), by varying DCB specimen dimensions and various cohesive zone properties, were investigated. Three values for inter-laminar strength  $t^o$  and two values for critical fracture energy  $G_I^C = G_{II}^C = G^C$  are used to provide range of cohesive zone properties. The configurations also have different rotation angles applied to the top beam, in a magnitude enough to reach steady state crack propagation.

Uniform and non-uniform meshes are modelled for the different configurations in which element sizes are chosen to ensure at least 10 elements in a damage cohesive zone. There were only three configurations which have a lower number of elements in damage zone than 10 (6 being the lowest). A uniform mesh (throughout each beam) is used in configurations where a damage zone is larger and a non-uniform in configurations where it is smaller in order to rationalise number of elements in a model and CPU usage in a simulation. The finest mesh is used in a zone around the initial crack tip (a beginning of a cohesive zone) with twice the zone length after the tip (in a cohesive zone) than in front of the tip.

### 3. LOCAL PARTITIONING: INTEGRATION OF ENERGY GOING INTO COHESIVE ELEMENT

The energy calculation method is based on a local approach and energy going into a cohesive element in a delamination process is monitored and calculated by numerical integration of output values for each element integration point. The integrals for mode I and mode II energy release rates are:

$$G_I = \int_0^{\delta_{nm}} \sigma d\delta_n, G_{II} = \int_0^{\delta_{sm}} \tau d\delta_t, \quad \dots(3)$$

where  $\delta_{nm}, \delta_{sm}$  are maximum (final) opening and shearing displacements of the cohesive elements,  $\delta_n, \delta_t$  are opening and shearing displacements of the cohesive elements and  $\sigma, \tau$  are normal and shear stresses (tractions  $t_n$  and  $t_s$ ). Values for two integration points in a cohesive element are averaged to obtain a final value for an element. More details about implementation of the method in Abaqus can be found in [1].

#### 4. NUMERICAL SIMULATION RESULTS, COMPARISON TO ANALYTICAL SOLUTIONS AND DISCUSSION

Eighteen Abaqus simulations of delamination in FRMM test were performed (table 1) using previously described test and simulation configurations. The simulation accuracy is evaluated by monitoring reaction moment in the upper beam boundary, where rotation is applied (more details in [1]), and analysing deviations of calculated total fracture energy release rates from prescribed critical fracture energy values. In all simulations, constant trend of reaction moment after delamination onset was registered, and the highest energy error calculated was 1 %. The length of damage zones (table 1) formed ahead of crack tip was monitored and growing trend is observed with increase in critical fracture energy ( $G$ ) and beam height ratio ( $h_1/h_2$ ) increase, and decrease in inter-laminar strength ( $t$ ). Damage zones had nearly constant length through delamination process, as expected for steady-state crack propagation.

Table 1. Overview of FE model configurations and simulation results

$\frac{h_1}{h_2}$	$G$ [ $\frac{J}{m^2}$ ]	$t$ [MPa]	Rotation angle $\frac{rad}{100}$	Mesh		Cohesive element size (the finest) (mm)	Length of fine mesh zone (mm)		Initial damage zone (mm)	Propagation length (mm)	
				Elements number	Uniform		Ahead cohesive zone	In cohesive zone		Crack	Damage
0.1	200	20	13	344x81	NO	0.2	10	20	4.4	10.2	14.4
0.1	200	45	13	344x81	NO	0.2	10	20	2.2	11.6	13.6
0.1	200	95	13.5	394x97	NO	0.1	5	10	0.8	3.9	4.7
0.1	3000	20	53	344x81	NO	0.2	10	20	11.2	9.6	20.8
0.1	3000	45	49.9	344x81	NO	0.2	10	20	6	8.6	14.6
0.1	3000	95	48	344x81	NO	0.2	10	20	3.6	7.2	10.8
1	200	20	15	344x22	NO	0.2	10	20	4.6	11	15.6
1	200	45	15	344x22	NO	0.2	10	20	2.2	12.4	14.4
1	200	95	13.5	394x26	NO	0.1	5	10	0.8	3.9	4.7
1	3000	20	63	300x16	YES	0.4	/	/	16.8	12.8	29.6
1	3000	45	57	344x22	NO	0.2	10	20	7.8	7.8	15.6
1	3000	95	49.5	344x22	NO	0.2	10	20	3.8	4.4	8.2
10	200	20	40	344x14	NO	0.2	10	20	4.8	33	37.5
10	200	45	37.6	344x14	NO	0.2	10	20	2	9.4	11.4
10	200	95	37.1	394x16	NO	0.1	5	10	0.6	3.8	4.4
10	3000	20	149.7	120x3	YES	1	/	/	20	11	30
10	3000	45	147	300x8	YES	0.4	/	/	8.8	8	16.4
10	3000	95	146	344x14	NO	0.2	10	20	3.8	9.6	13.4

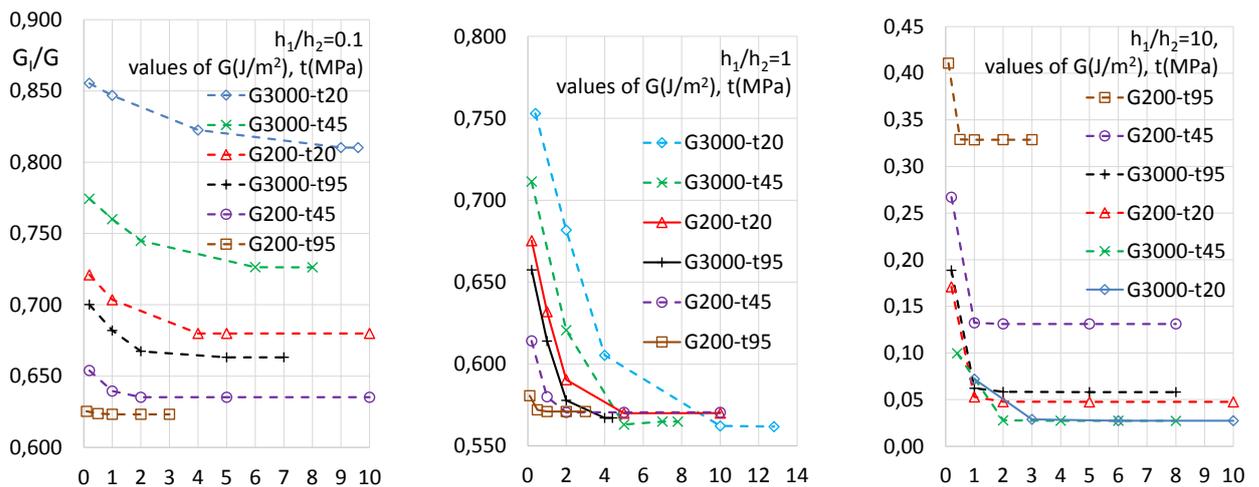


Figure 2. Mode-mixity ratio  $G_I/G$  change with crack propagation (mm) for different configurations

Components of the fracture energy (3) are calculated for cohesive elements along the crack propagation direction in order to monitor change of mode-mixity with crack propagation and get converged values. Mode-mixity is represented with the ratio of mode I energy release rates component to the total energy,  $G_I/G$  (Figure 2). Size of the observed converging zones seems to be dependent on the damage zone sizes.

Extensive research [5],[5] has been carried in the last 30 years in area of mixed mode fracture in layered materials producing several analytical partitioning theories that are used to

predict mixed mode partitions as a function of geometry ( $h_1/h_2$ ), but there is still much confusion around their validity and application in practice. Mode partitioning calculated from the simulation results are compared (Figure 3) with the Williams analytical solution [2], based on a global approach using beam theory, and Hutchinson and Suo semi-analytical solution [3], based on linear elastic fracture mechanics (LEFM). Dependency of a mode partitioning on cohesive zone properties is observed, in a range between the two analytical solutions, confirming the findings reported in [5]. The dependency is more pronounced in asymmetric beam geometries ( $h_1/h_2 \neq 1$ ), while for the symmetric case ( $h_1/h_2=1$ ) mode-mixity is nearly independent on cohesive properties, as suggested by all analytical solutions. Obtained numerical solutions from configurations with higher fracture energy and lower inter-laminar strength are closer to Williams, and vice versa to Hutchinson and Suo solution, therefore the two analytical solutions seem to form the upper and lower bound of the mixed mode partitioning solution.

## 5. CONCLUSIONS

Mode-mixity dependence on cohesive model properties is observed in FE simulations of FRMM test, opposite to existing analytical partitioning solutions. The implemented cohesive zone model had some limitations but it is believed that the dependence is general. In order to obtain the more accurate property-dependent partitioning solution, further investigations are needed.

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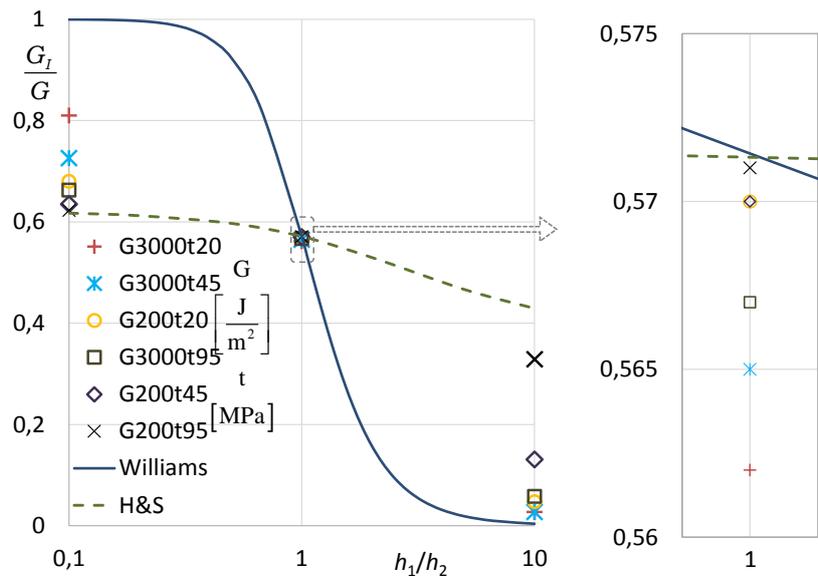


Figure 3. Mode-mixity ratio  $G_I/G$  in FRMM test obtained numerically (for different geometries and cohesive zone properties) and analytically (Williams[2] and Hutchinson and Suo [3] solutions)