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ANALYSIS OF VELOCITY AND TEMPERATURE PROFILES ACROSS VERTICAL FLAT SURFACE OF RADIATORS DURING NATURAL CONVECTION

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ABSTRACT
Heat transmission through a vertical flat surface of a radiator is strongly influenced by temperature and velocity profiles along such surface. The velocity and temperature profiles correspond to natural convection and depend on Grashof and Prandtl number. In order to obtain velocity and temperature profiles of current natural convection problem, continuity equation, momentum equation and energy equation have been utilized. These partial differential equations have been transformed into two ordinary differential equations of third degree by means of independent dimensionless variables and of the stream function. Such equations are solved numerically and the results are compared with results of an analytical solution.
Keywords: Natural convection, Velocity and temperature profiles, Boundary layer

1. INTRODUCTION
The movement of fluid during natural convection is caused by buoyancy forces, which, on the other hand, are created as result of density variation across fluid layers up to the distance to which the impact of wall temperature is observed. If the surface temperature of the vertical plate is higher than the temperature of the surrounding environment, the temperature of the fluid close to the vertical plate increases, while its density decreases [1]. As a result of the changing density of the fluid layers in the horizontal direction, warmer layers close to the hot wall move upwards, allowing in that way to be replaced by colder layer with higher density and far from the hot wall. On the other hand, the temperature of the fluid layers contacting the warm plate is equal to the plate temperature, while for other layers in horizontal direction it gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface [2].

2. NATURAL CONVECTION IN VERTICAL PLATE
Heat transfer by natural convection often occurs in many physical problems and engineering applications. However, solving of governing equations describing this problem is not straight forward process, because of the nonlinearity of such equations. Therefore, such equations usually are solved numerically and under specific assumptions for few cases, also, analytically. The basic difficulty in solving of problems related to natural convection lies in necessity of simultaneous solving of velocity and temperature profiles.

2.1 Numerical methods

Boundary Layer Equations

Continuity equation $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \qquad (1)$$

Momentum equation $$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}=\beta g\left(T-T_{\infty}\right)+\upsilon\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

Energy equation $$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\alpha\frac{\partial^2 \theta}{\partial y^2} \qquad \text{where } \theta \text{ is defined as } \theta=\frac{T-T\infty}{T_s-T_{\infty}} \qquad (3)$$

Stream function $$u=\frac{\partial \psi}{\partial y} \qquad v=-\frac{\partial \psi}{\partial x} \qquad \psi=4\nu\left(\frac{Gr_x}{4}\right)^{1/4} f(\eta) \qquad (4)$$

Similarity variables introduced $$\eta=\left(\frac{Gr_x}{4}\right)^{1/4}\frac{y}{x}; \quad \theta(x,y)=\theta(\eta); \quad \frac{df}{d\eta}=\frac{u\cdot x}{2\nu\sqrt{Gr_x}} \qquad (5)$$

Grashof number $$Gr_x=\frac{\beta g\left(T_s-T_{\infty}\right)x^3}{\upsilon^2} \qquad (6)$$

With the replacement of the equations (4), (5), (6) in equations (2) and (3) following ordinary differential equations can be obtained [1,3].

$$\frac{d^3 f}{d\eta^3}+3f\frac{d^2 f}{d\eta^2}-2\left(\frac{df}{d\eta}\right)^2+\theta=0 \qquad (7)$$

$$\frac{d^2\theta}{d\eta^2}+3\,\mathrm{Pr}\cdot f\frac{d\theta}{d\eta}=0 \qquad (8)$$

Boundary conditions for velocity and temperature are defined as follows:

$$f(\eta=0)=0; \qquad \frac{df}{d\eta}(\eta=0)=0; \qquad \frac{df}{d\eta}(\eta\to\infty)=0; \qquad \theta(\eta=0)=1; \qquad \theta(\eta\to\infty)=0$$

The differential equations (7) and (8) for the defined boundary conditions can be solved with the help of Matlab. Corresponding results for the dimensionless velocity and temperature profile of fluid layers adjacent to the vertical flat surface are presented in Figs. 1 and 2.

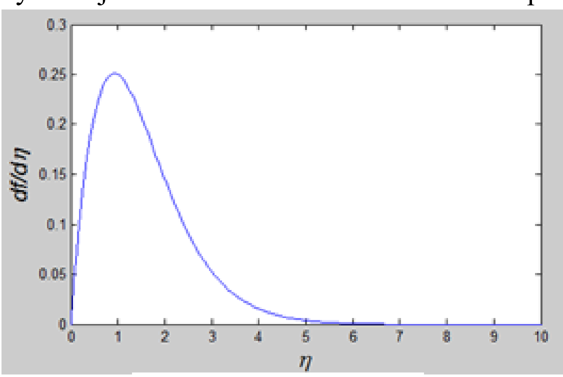

Fig.1 Velocity profile

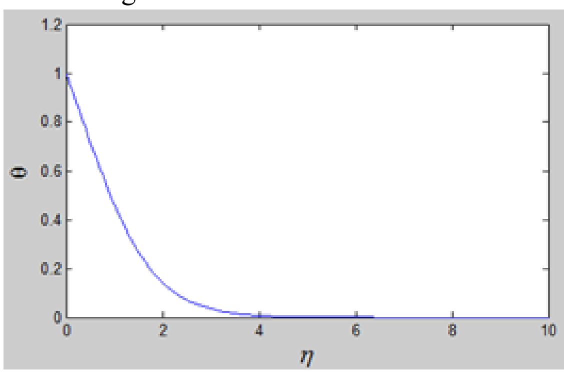

Fig.2 Temperature profile

2.2 Analytical methods

By integrating the momentum equation and energy equation following integral boundary layer equations for energy and momentum can be derived [2]

Energy Equation: $$\frac{d}{dx}\int_{y=0}^{\delta_T(x)} u\left(T-T_{\infty}\right)dy=\left(\frac{k}{\rho c_p}\right)\left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (9)$$

Momentum Equation: $$\frac{d}{dx}\int_{y=0}^{\delta_v(x)} u^2 dy=\left(\frac{\mu}{\rho}\right)\left.\frac{\partial u}{\partial y}\right|_{y=0}+\int_{y=0}^{\delta_T(x)} g\beta\left(T-T_{\infty}\right)dy \qquad (10)$$

In order to solve equations (9) and (10) velocity and temperature profiles are required.
For the current study following equation is assumed to apply for the velocity profile:

$$u(x,y) = a_0(x) + a_1(x)y + a_2(x)y^2 + a_3(x)y^3 \tag{11}$$

The boundary conditions corresponding to velocity are defined as follows:

$$u(x,0)=0; \quad u(x,\delta)\approx 0; \quad \frac{\partial u(x,\delta)}{\partial y}\approx 0; \quad \frac{\partial^2 u(x,0)}{\partial y^2} = -\frac{\beta\cdot g}{\upsilon}\cdot(T_s - T_\infty)$$

After replacement of boundary conditions in equation (11), it can be obtained:

$$u(x,y) = u_0(x)\cdot\frac{y}{\delta}\cdot\left(1-\frac{y}{\delta}\right)^2 \quad \text{where} \quad u_0(x) = \left[\frac{\beta\cdot g}{4\upsilon}\cdot(T_s - T_\infty)\right]\cdot\delta^2 \tag{12}$$

For the temperature profile following equation is assumed to apply:

$$T(x,y) = b_0(x) + b_1(x)y + b_2(x)y^2 \tag{13}$$

The boundary conditions for the temperature are as follows:

$$T(x,0)=T_s; \quad T(x,\delta)\approx T_\infty; \quad \frac{\partial T(x,\delta)}{\partial y}\approx 0$$

After replacement of boundary conditions in equation (13), one can obtain:

$$T(x,y) = T_\infty + (T_s - T_\infty)\cdot\left(1-\frac{y}{\delta}\right)^2 \quad \text{where} \quad \theta = \frac{T(x,y)-T_\infty}{T_s - T_\infty} = (1-\frac{y}{\delta})^2 \tag{14}$$

The maximum value of the velocity is achieved for ratio y/δ=1/3.
The velocity and temperature profile depending on the ratio y / δ are present in the figures 3 and 4.

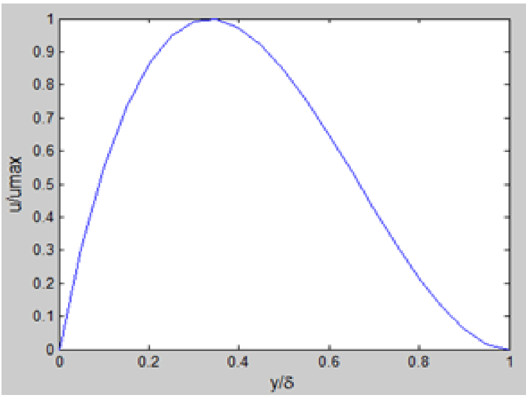

Fig.3 Velocity profile

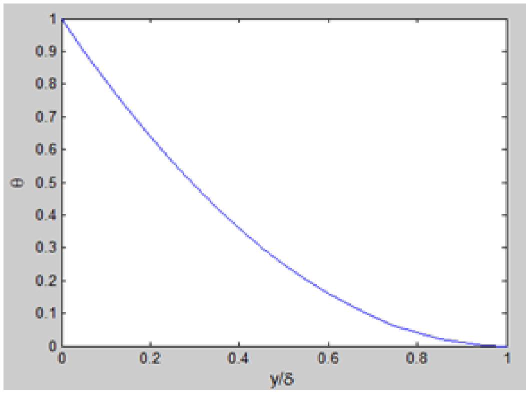

Fig.4 Temperature profile

3. COMPARISONS OF VELOCITY AND TEMPERATURE PROFILES

3.1 Comparison of velocity

The comparison is made for reference point in relative variables which corresponds to maximum value of the velocity profile. Simultaneously for the same point is made also the comparison of temperature profiles of boundary layer.
Boundary layer thickness obtained analytically [1].

$$\delta(x) = Bx^n \text{ where } B = 3.936\left(\frac{20}{21}+\Pr\right)^{\frac{1}{4}}\Pr^{-\frac{1}{2}}\left(\frac{\beta g(T\ -T_\infty)}{\nu^2}\right)^{-\frac{1}{4}}; \ n=\frac{1}{4}; \qquad \Pr=1.0$$

$$\delta = Bx^{\frac{1}{4}} = 4.653\frac{x}{Gr_x^{0.25}} \quad \text{respectively} \quad x = \delta(Gr_x)^{0.25}\cdot\frac{1}{4.653}$$

The value x replacement in dimensionless variable η gives:

$$\eta = \left(\frac{Gr_x}{4}\right)^{\frac{1}{4}}\frac{y}{x} = 3.29\frac{y}{\delta} \quad \text{where for } \frac{y}{\delta}=\frac{1}{3}, \text{ gives } \eta = 1.097$$

From figure 1, one can see that maximal velocity is obtained for $\eta = 0.99$, which means the difference between the maximal velocity related abscise, derived analytically and numerically (graphically) is 10.8%.
Velocity profile of boundary layer solved analytically [2]:

$$u_0(x) = Ax^m \quad ; \; A = 5.164\upsilon\left(\frac{20}{21} + \Pr\right)^{-\frac{1}{2}}\left(\frac{g\beta(T - T_\infty)}{\upsilon^2}\right)^{\frac{1}{2}} \text{ where } m = \frac{1}{2}$$

$$u_0(x) = 3.696\upsilon\sqrt{Gr_x}\,\frac{1}{x} \text{ where } x = \frac{3.696\upsilon\sqrt{Gr_x}}{u_0(x)}$$

Based in equation (12) for *y/δ*= 1/3 maximal value of the velocity is $u_{\max} = \frac{4}{27}u_0(x)$ and hence following calculation applies for the dimensionless velocity:

$$\frac{df}{d\eta} = u\frac{x}{2\upsilon\sqrt{Gr_x}} = \frac{u}{2\upsilon\sqrt{Gr_x}} \cdot \frac{\upsilon \cdot 3.696\sqrt{Gr_x}}{u_0(x)} = \frac{u \cdot 3.696}{2 \cdot \frac{27}{4}u_{\max}} = 0.274$$

On the other hand from figure 1, one can see that maximal dimensionless velocity is *df/dη*=0.251, which means the difference between analytical and numerical (graphical) solution is 9.1%.
Comparison of numerical and analytical solutions, based on described procedure, for various Pr is presented in Table 1.

Table 1. Comparison of numerical and analytical solutions for various Pr

Pr	η_{num}	η_{anal}	*diff [%]*	$(df/d\eta)_{num}$	$(df/d\eta)_{anal}$	*diff [%]*	Θ_{num}	*diff [%]*
0.72	1.02	1.243	21.8	0.276	0.296	7.2	0.51	18.6
1.0	0.99	1.097	10.8	0.251	0.274	9.1	0.47	9.3
2.0	0.83	0.86	3.6	0.202	0.223	10.4	0.44	6.9

3.2. Comparison of temperature

Based on temperature profile derived from numerical solution (fig. 2), for η=1.097 which corresponds to maximal dimensionless velocity, the value of relative temperature is θ=0.43, while the analytical solution obtained for *y/δ*= 0.33 (corresponding to maximal velocity) results in relative temperature θ=0.47 (fig.4). It means in case of temperature profile the difference between analytical and numerical (graphical) solution is 9.3%.

4. CONCLUSIONS

Current paper provides a comparison of the solutions for the velocity and temperature profiles for the heat transfer by natural convection, obtained by numerical and analytical methods. Comparisons of velocity profiles, for Pr=1.0 resulted in a difference of 10.8 % of the dimensionless variable while the difference noted for the maximal velocity obtained numerically and analytically was 9.1%.
Current analysis further shows that the difference between the relative temperatures obtained numerically and analytically is 9.3 %. Further, based on results presented in table 1 it can be concluded that for Pr>1 the differences between numerical and analytical solutions for velocity and temperature profiles decreases while the opposite is through for Pr<1.

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