DESIGN OF THE DOOR OPENING MECHANISM WITH COMPLEX STRUCTURE

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ABSTRACT
In this paper a method for dimensional kinematic synthesis of complex mechanisms with high class kinematic groups is proposed. The basic idea of this method is to form a mathematical model of the high class kinematic group by decomposing it to second class kinematic group modules. A system of equations is obtained from the kinematic constrains in the constituting modules and has the form of \( f_i(x) = 0 \). Dyadic modules are combined thus ensuring that all possible solutions of mechanism configurations are obtained. Functional constrains are given by the precision positions and can also be written in the form \( F_i(x) = 0 \). The objective function is defined as a sum of \( f^2_i(x) = 0 \) and \( F^2_i(x) = 0 \) and the synthesis problem is solved as a nonlinear least-squares curve fitting problem.

Keywords: design, mechanism synthesis, high class kinematic group

1. INTRODUCTION
Complex structure mechanisms with high class kinematic groups have significant kinematic and dynamic potential and they present an immense technological pool for new ingenious design solutions [1], [2]. However, they could rarely be found in practical application. The main reason for this situation is their complex structure which, on one side, gives them superior kinematic and dynamic capabilities but, on the other side, significantly complicates their analysis and, even more their dynamical synthesis.

2. MECHANISMS SYNTHESIS ISSUES
Methodology of precision position dimensional synthesis is well developed for both planar and spatial mechanisms but only for mechanisms with a simple structure: in literature solutions can be found for practically applicable cases and types of synthesis of two link open kinematic chains - dyads, as well as the synthesis of the three link open kinematic chains - triads with rotational joints only [3]. Solutions of several cases of the synthesis of four link mechanisms can be found in [4]. The synthesis of six link mechanisms with a simple structure is presented in [5]. In [6] a case of synthesis of mechanisms with third class kinematic group using the modular approach is presented, moduls are constituting dyads and tryads. Values of design parameters are obtained by solving a system of nonlinear equations which describe the position of constituting modules. Numerical methods are commonly applied for solving this system of equations. The application of these methods has many problems:

- For the method convergence, the starting values of variables must be close to exact solutions.
- Absence of convergence exists when the system is close to singular positions.
- Non-linear equations always have multiple solutions, the number of which is unknown. This means that there can exist several possible mechanism configurations that are the solution of the synthesis problem.

The final problem is very important from the designers point of view. The existence of multiple solutions which satisfy the prescribed requirements gives the designer the possibility to choose among them and, in a certain way, a possibility of additional optimization. A method of synthesis which can show all possible solutions for the given constrains would present an enormous aid in the process of mechanism design.
3. SYNTHESIS OF THE MECHANISM WITH THE HIGH CLASS KINEMATIC GROUP

In Fig. 1, a mechanism for door opening is presented. It is a complex mechanism with a third class kinematic group. It has to move the door - link 6, through a series of prescribed positions which are given with the position of the end point of the door G and its angle $\varphi_6$.

![Fig.1. Door opening mechanism](image1)

![Fig.2. Vectors describing position of the RR-RT-TR third class kinematic group](image2)

The door mechanism is a complex mechanism consisting of two kinematic groups (Fig. 1.): the first class kinematic group - input link 2 and the third class kinematic group of the RR-RT-TR type – links 3, 4, 5 and 6. The third class kinematic group is connected to the input link at point B.

The basic idea of the method presented in this paper is to make the mathematical model of the high class kinematic group using second class kinematic groups (dyads) - modules. By combining modules, any type of high class kinematic group can be formed. This way of forming and combining modules into a system ensures that all possible solutions for the high class kinematic group position are found. Types of modules and their application for mechanism analysis are presented in [7]. Using that principle, the third class kinematic group can be modelled in following way:

First, the position of point F is calculated as:

$$\vec{r}_F = \vec{r}_3 + \vec{r}_5$$  \hspace{1cm} (1)

Now, a dyad of type RRT (links FG and HG) is formed:

$$\vec{r}_F + \vec{r}_6 = \vec{r}_5 + \vec{r}_7 = \vec{r}_G$$  \hspace{1cm} (2)

From this equation distance $r_5$, angle $\varphi_{61}$ and the position vector $\vec{r}_G$ can be derived.

Links GC and BC form a RTR dyad:

$$\vec{r}_G + \vec{r}_6 = \vec{r}_3 + \vec{r}_4 = \vec{r}_C$$  \hspace{1cm} (3)

From this equation distance $CG$, angle $\varphi_{62}$ and the position vector $\vec{r}_C$ can be derived.

Finally, a dyad of type RRR (links DF and EF) is formed:

$$\vec{r}_D + \vec{r}_6 = \vec{r}_4 + \vec{r}_7 = \vec{r}_F$$  \hspace{1cm} (4)

From this equation distance angles $\varphi_{63}$ and $\varphi_4$ and the position vector $\vec{r}_F$ can be derived.

When written in scalar form, eqs. (1)-(4) give:

$$x_F = x_F + \frac{EF}{\sin \varphi_4} \cos \varphi_4$$  \hspace{1cm} (5)

$$y_F = y_F + \frac{EF}{\sin \varphi_4} \sin \varphi_4$$  \hspace{1cm} (6)

$$\overline{HG} = -\left( (x_H - x_F) \cos \varphi_3 + (y_H - y_F) \sin \varphi_3 \right) \pm \sqrt{\left( (x_H - x_F) \cos \varphi_3 + (y_H - y_F) \sin \varphi_3 \right)^2 - \left( x_H - x_F \right)^2 + \left( y_H - y_F \right)^2}$$  \hspace{1cm} (7)

$$x_G = x_H + \frac{\overline{HG}}{\cos \varphi_3} \cos \varphi_3$$  \hspace{1cm} (8)

$$y_G = y_H + \frac{\overline{HG}}{\cos \varphi_3} \sin \varphi_3$$  \hspace{1cm} (9)

$$x_B = x_A + \frac{AB}{\cos \varphi_2} \cos \varphi_2$$  \hspace{1cm} (10)

$$y_B = y_A + \frac{AB}{\cos \varphi_2} \sin \varphi_2$$  \hspace{1cm} (11)
\[
G = \sqrt{\left( (x_h - x_g)^2 + (y_h - y_g)^2 \right) - BC^2}
\]  
(12)

\[
\varphi = \text{arctan} \left( \frac{y_h - y_g}{x_h - x_g} \right) \pm \arccos \left( \frac{\left( (x_h - x_g)^2 + (y_h - y_g)^2 \right)^{1/2} + \frac{G ^ 2 - BC ^ 2}{2 \left( (x_h - x_g)^2 + (y_h - y_g)^2 \right) G}}{G} \right)
\]  
(13)

\[
x_c = x_h + G \cos \varphi
\]  
(14)

\[
y_c = y_h + G \sin \varphi
\]  
(15)

\[
x_d = x_g + G \cos \varphi
\]  
(16)

\[
y_d = y_g + G \sin \varphi
\]  
(17)

\[
\varphi_3 = \text{arctan} \left( \frac{y_d - y_f}{x_d - x_f} \right) \pm \arccos \left( \frac{DF ^ 2 + \left( (x_h - x_g)^2 + (y_h - y_g)^2 \right) - EF ^ 2}{2DF \left( (x_h - x_g)^2 + (y_h - y_g)^2 \right)} \right)
\]  
(18)

\[
x_f' = x_d + \frac{DF \cos \varphi_3}{2DF} \]  
(19)

\[
y_f' = y_d + \frac{DF \sin \varphi_3}{2DF} \]  
(20)

The sign ± in eqs. (7), (13) and (18) means that dyads can assume two possible configurations, leading to the conclusion that it is possible to assemble mechanism in 2^2*2*2=8 configurations i.e. there are 8 possible solutions to the synthesis problem.

Design parameters that have to be determined are:

- link lengths: EF, GF, AB, BC, DF,
- positions of fixed points A and E: x_E, y_E, x_A, y_A,
- position of input link 2: \( \varphi_2 \)
- position of one of the output links, in this case link 4 is chosen: \( \varphi_4 \)

The position of point H and the angle of the sliding guide for link 5 can be adopted and are not design parameters. Points F and F' have to be at the same place in the real mechanism so the next two equations present conditions which design parameters have to satisfy in order for it to be possible to assemble the mechanism and they are called kinematic constraints:

\[
f_1 = x_f' - x_f = 0
\]  
(21)

\[
f_2 = y_f' - y_f = 0
\]  
(22)

On the other side, the mechanism has to fulfill its function i.e. to pass through precise positions so following constraints, called functional constraints, can be written:

\[
f_3 = x_g - x_f \text{ for position } i = 0
\]  
(23)

\[
f_4 = \varphi_3 - \varphi_4 \text{ for position } i = 0
\]  
(24)

The problem structure depending on the number of precision positions is presented in Table 1. In cases when the number of precision positions is less than 5, there are more unknowns (design parameters) than equations (constraints) so values for some of the parameters have to be adopted in order to solve the problem. If, after adoption, the number of equations equals the number of unknowns, there exists a finite number of solutions. If 5 or more precision positions are given, the number of equations is larger than the number of unknowns and the problem has to be solved using optimization techniques – a procedure will search for a solution that satisfies the constraints in the best possible way.

Since all the constraints are expressed in the form \( f_i = 0 \) it is most efficient to use the least-squares method. The optimization problem is defined as:

\[
x = \min \left( f_1^2 (x) + f_2^2 (x) + ... + f_n^2 (x) \right)
\]  
(25)

<table>
<thead>
<tr>
<th>Number of precision positions</th>
<th>Number of constraints</th>
<th>Number of design parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>19</td>
</tr>
</tbody>
</table>
The lower and upper bounds of design parameters $x$ are given:

$$a < x < b$$

Vector $x$ is a vector of design parameters (N is a number of the precision positions):

$$x = [E, F, AB, BC, DF, x_E, y_E, x_A, y_A, \varphi_{21}, \varphi_{41}, \ldots, \varphi_{2N}, \varphi_{4N}]$$

(27)

By conducting an optimization procedure for all combinations of + and - in eqs. (7), (13), (18), all possible solutions - mechanism configurations will be obtained.

4. EXAMPLE

The synthesis of the mechanism for door opening (kinematic scheme is presented in Fig.1.) has to be performed. The position of point H and the angle of the sliding guide for slider 5 are adopted: H(0,0) and $\varphi_x=180^\circ$. The precision positions are given in Table 2. As a result of the synthesis several mechanism configurations were obtained. Solution with $++-$ combination is presented. The obtained values for the design parameters and positions of point G and the door angle are presented below.

| Table 2. Precision positions | $EF=0,197m$, $GF=0,113m$, $AB=0,379m$, $BC=0,106m$, $DF=0,704m$, $x_E=-0,127m$, $y_E=0,249m$, $x_A=-0,182m$, $y_A=0,162m$, $\varphi_{21}=29,6^\circ$, $\varphi_{41}=185^\circ$, $\varphi_{2R}$ $\varphi_{4R}$ $x_G$ $\varphi_{door}$
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>position</td>
<td>$x_G$</td>
</tr>
<tr>
<td>1</td>
<td>-0.45m</td>
</tr>
<tr>
<td>2</td>
<td>-0.43m</td>
</tr>
<tr>
<td>3</td>
<td>-0.41m</td>
</tr>
<tr>
<td>4</td>
<td>-0.38m</td>
</tr>
<tr>
<td>5</td>
<td>-0.35m</td>
</tr>
</tbody>
</table>

Table 3. Obtained door positions

<table>
<thead>
<tr>
<th>position</th>
<th>$\varphi_2$</th>
<th>$x_G$</th>
<th>$\varphi_{door}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29,6$^\circ$</td>
<td>-0.435m</td>
<td>20$^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>59,0$^\circ$</td>
<td>-0.431m</td>
<td>40$^\circ$</td>
</tr>
<tr>
<td>3</td>
<td>81,1$^\circ$</td>
<td>-0.410m</td>
<td>60$^\circ$</td>
</tr>
<tr>
<td>4</td>
<td>96,8$^\circ$</td>
<td>-0.379m</td>
<td>80$^\circ$</td>
</tr>
<tr>
<td>5</td>
<td>99,6$^\circ$</td>
<td>-0.350m</td>
<td>90$^\circ$</td>
</tr>
</tbody>
</table>

By comparing results from Table 3. with the prescribed values in Table 2. it can be seen that the only significant difference exists at the first position of point G - about 4%. The differences for all other positions are negligible. The mechanism is presented in Fig.3. in all working positions.

5. CONCLUSION

The presented method for the synthesis of complex mechanisms is based on the modular approach for modeling high class kinematic group. It has two main advantages: first, the constraints are modeled by equations in the form $f_i(x)=0$ which are suitable for the use of the least-square optimization method and the second, even more important advantage, it gives all possible solutions for mechanism configurations that satisfy the required functional constraints.

8. REFERENCES


