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PREDICTION ANALYSIS OF RECEIVABLES USING MARKOV MODEL IN PUBLIC COMPANY FOR WATER SUPPLY

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ABSTRACT
In making business decisions Markov model, being a tool in predicting the situation in which one can find an economic phenomenon/system in the future, plays very important role. Stohasticity is one of the dominant characteristics of Markov models. Prilikom prediction, it is very important to monitor the movement of economic indicators in earlier periods, in order to define the correct probabilities of the system transition from one state to another. In the paper Markov model is applied in predicting the receivables collectibility from households, in the case of the company "Water and Sewerage" LLC Podgorica.
Keywords: Markov processes, stochastic probability, prediction, optimal control

1. INTRODUCTION
Economic systems belong to a group of social systems. Complexity is an important characteristic of these systems. The management and development of most economic systems influence the decisions that contain elements of uncertainty and where each event changes the probability of the subsequent events. The economic phenomenons are stochastic ones. Therefore, in analyzing and forecasting these phenomenons, it is not possible to determine their exact future values. Markov models, belonging to quantitative ones, can be used in studying and predicting future economic activities. Markov models are stochastic ones. In these models future event probability predictions are based on the information on experienced current probability [1]. In this paper, we demonstrate the possibility to use Markov model in predicting future receivables in case of "Water and Sewerage" LLC Podgorica, based on knowledge of the current situation and probability of change in this situation during prediction time.

2. ASSUMPTIONS OF MARKOV MODEL
Markov model is a mathematical one often used in the analysis of complex systems in general, and economic systems, in particular. The basic concept of Markov model arises from the probability theory. In Markov model, in order to predict the state of a system in the future, it is necessary, firstly, to know the initial condition of the system. Then it is necessary to determine the probability of transition of the system from the initial state into another state during the observed time interval. The probabilities of the change of system status are presented in the probability matrix, whose elements are the individual probabilities relating to modification of the system from one state to another.
The assumptions of Markov model are:
1) There is a finite number of possible states of the system; 2) Status of the system in a period depends solely on its status in the immediately preceding period; 3) The system size and the number of system elements remain unchanged during the period for which the prediction is carried out; 4) Transition probability matrix is stationary, i.e. it does not change during the forecasting period. [1]

Also, it should be pointed out that at one point of time the system can not be found simultaneously in two different states. If t$s(t)$ denotes the state vector of a system in the period t, and $s_1(t), s_2(t),..., s_n(t)$ denote possible likelihood that the system will be found in one of n-posible conditions then the system state in a given period of time, can be stated as $s(t) = (s_1(t), s_2(t),..., s_n(t))$.

3. MARKOV TRANSITIVE MATRIX

In a given period of time, the system can move from the current (i-th) state to the other (j-th) state, as shown by the transitive probabilities. Transitive probability (p_{ij}) denotes the conditional probability that indicates that the system of the i-th state in period t, moves into the j-th state in period (t + 1). As the system can be found in n different states, all the probabilities of system state changes are represented in a Markov transitive matrix. The general form of Markov transitive matrix is given as follows

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}, \quad \sum p_{ij} = 1, \ \ 0 \le p_{ij} \le 1, \ \ i, j = 1,2,...n$$

Elements located on the main diagonal of Markov transitive matrix indicate the probability that the system will not change the state from the previous period after the expiry of the time interval, while the other probabilities indicate the chance of achieving changes in state of the system within the observed time interval. [1]

4. MARKOV MODEL FOR DETERMINING THE FINAL BALANCE OF COMPANY'S RECEIVABLES

Business policy of the company largely depends on its customers. The information on receivables collection is a very important information when making decisions. It is Markov model precisely that gives the possibility to company to predict the collectability of certain categories of receivables. Based on experience all receivables can be divided into four categoris [3]:

State 1 (S_1): Collected receivables
State 2 (S_2): Written off receivables
State 3 (S_3): Receivables with a term of payment up to 30 days
State 4 (S4): Receivables with a term of payment period up to 30 to 90 days.

Collected and written off receivables are the final states of receivable because they can not change into another state. The remaining two receivable states are considered to be normal states, given their changeable nature since they can switch either to collected or written-off receivables.

To formulate a Markov model in order to use it in the analysis and forecasting of receivables, it is necessary to firstly define the Markov transitive probability matrix. As the first two of receivables categoris are at final state, the probability of keeping them in their present state is equal to 1, while the probability of transition of these categories into some of the remaining states equals 0. However, it is the most important to formulate the probability of transition from the third and fourth category into some other receivable state, i.e. into a normal state. These probabilities are determined usually on the basis of previous experience. Hence, Markov transitive probability matrix is

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} p_{31} & p_{32} \\ p_{41} & p_{42} \end{bmatrix},$$

$$B = \begin{bmatrix} p_{33} & p_{34} \\ p_{43} & p_{44} \end{bmatrix}; \; tj. \; M = \begin{bmatrix} I & 0 \\ A & B \end{bmatrix}$$

The fundamental matrix (F) can now be defined as follows:

$$F = (I - B)^{-1}.$$

The probabilities of transition from the third and fourth receivable state into the first and second state are obtained on the basis of the following relations

$$K = FA = \frac{1}{(1-p_{33})(1-p_{44}) - p_{34}p_{43}} \begin{bmatrix} 1-p_{44} & p_{34} \\ p_{43} & 1-p_{33} \end{bmatrix} \begin{bmatrix} p_{31} & p_{32} \\ p_{41} & p_{42} \end{bmatrix} = \begin{bmatrix} k_{31} & k_{32} \\ k_{41} & k_{42} \end{bmatrix},$$

where elements of the first column of transitive probability matrix indicate the probabilities of moving receivables from the third state into the first or second final state, and the elements of other columnes indicate the probabilities of receivables transition from the fourth state into the first or second receivable state, i.e. final state. If the state of receivables in a single point in time is presented by a vector

$$Q_p = (q_3, q_4)$$

where q_3 indicates amount of the receivables in state S_3 state and q_4 amount of the claims in state S_4, the total amount of receivables to be collected or written off, is obtained as the product Q_pK, i.e.

$$Q_c = Q_p K = (q_3, q_4) \begin{bmatrix} k_{31} & k_{32} \\ k_{41} & k_{42} \end{bmatrix} = (q_{1c}, q_{2c})$$

where q_{1C} denotes the amount of the of receivables in third and fourth state of receivables to be collected, and q_{2c} is the amount of the third and fourth state of claims to be canceled (written off).

5. EMPIRICAL ANALYSIS OF PREDICTIONS OF RECEIVABLES COLLECTION IN "WATER AND SEWERAGE" LLC PODGORICA

Receivables from distributive consumers bring the most revenue to the company "Water and Sewage" LLC Podgorica. Their name comes from the fact that these group of consumers are connected to the distribution network. Furthermore, this group of consumers, and accordingly this type of receivables, are devided to two parts: (1) the households and (2) commercial consumers cosisting of a multitude of companies that conduct their business activities in Montenegro. Markov model can pretty much help in predicting the collection of claims. In this, the starting point is the determination of the status of claims. In this case, the total receivables from households can be divided into the following categories:

- Collected claims in 2014
- Written off claims in 2014
- Receivables from households with payment period of up to 30 days
- Receivables from households with payment period of 30 to 90 days

The next important step in defining a Markov transitive probability matrix is the determination of the probabilities of receivables transition from one state to the other. The experience with which the company has faced collection of these claims during 2014 is of great help in determining the transitive probabilities. The structure of receivables from households in 2014, in "Water and Sewerage" LLC Podgorica, is given in the following table:

Tabela 1: Receivables from households, in 2014

Quarter	Invoices	Collected	Not collected	% collected
I	1.355.485	921.338	434.147	67,97
II	1.478.304	954.030	524.274	64.53
III	1.656.967	1.048.765	608.202	63.29
IV	1.565.111	1.366.397	198.714	87.30

Markov transitive probability matrix (M), determined on the basis of experience and the matrix K, which contains the probability of transition of the third and fourth categories of claims into the first or second final state is as follows:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0{,}60 & 0{,}20 & 0{,}15 & 0{,}05 \\ 0{,}40 & 0{,}25 & 0{,}20 & 0{,}15 \end{bmatrix},$$

$$K = FA = \frac{1}{0{,}7129}\begin{bmatrix} 0{,}85 & 0{,}05 \\ 0{,}20 & 0{,}85 \end{bmatrix} \times \begin{bmatrix} 0{,}60 & 0{,}20 \\ 0{,}40 & 0{,}25 \end{bmatrix} = \begin{bmatrix} 0{,}74 & 0{,}26 \\ 0{,}64 & 0{,}36 \end{bmatrix}$$

The first column of the matrix K, indicates that the probability of collection of third category receivables is 74%, while 26% of these claims will be written off in the first quarter of 2015. Probability of collection of fourth category receivables is 64% and 36% of these claims will not be charged in the first quarter of 2015. These are very important information since, in the long term, receivable states S_3 and S_4 after a certain period must move into one of the two final states. In this way, the company acquired very important information on expected capital inflow during the next period.

6. CONCLUSION

The nature of economic phenomenons is such that it is impossible to precisely predict their future values. In this paper we have demonstrated that the Markov model can be used to determine the probability of collectability of certain receivable categories. The importance of use of Markov models arises from the fact that it helps decision makers to anticipate, to a large extent and with great precision, the movements of the observed phenomenon (receivables), in the future. Non-collected claims are problem of every company. For the purposes of this study, uncollected receivables are divided into receivables with maturities up to 30 days and receivables with maturities of 30 to 90 days. The rapid development of computer technology, which allows fast and easy solution to very complex problems of quantitative analysis as well as their growing use in various areas of management contributes significantly to expanding the application of quantitative models in the field of business management.

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