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AN OVERVIEW OF WAVELET BASED SIGNAL PROCESSING

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ABSTRACT
Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction. This paper introduces wavelets to the interested technical person outside of the digital signal processing field. The history of wavelets is described by beginning with Fourier transform, short time Fourier transform, continuous wavelet transform, discrete wavelet transform and comparing wavelet transforms with Fourier transforms. Then, an application for comparing wavelet transforms is performed for sinusoidals with additional noise and CWT is investigated.
Keywords: wavelets, wavelet transformation, signal processing

1. INTRODUCTION
The fundamental idea behind wavelet is to analyze according to scale. Indeed, some researchers feel that using wavelets means adopting a whole new mind-set or perspective in processing data [1]. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. This idea is not new. Approximation using superposition of functions has existed since the early 1800s, when Joseph Fourier discovered that he could superpose sines and cosines to represent other functions. However, in wavelet analysis, the scale that used to look at data plays a special role. Wavelet algorithms process data at different scales or resolutions. If it is looked at a signal (or a function) through a large "window," someone would notice gross features. Similarly, if it is looked at a signal through a small "window," small features would be noticed. This makes wavelets interesting and useful. With wavelet analysis, we can use approximating functions that are contained neatly in finite domains. Wavelets are well-suited for approximating data with sharp discontinuities [2].
The wavelet analysis procedure is to adopt a wavelet prototype function, called an analyzing wavelet or mother wavelet. Temporal analysis is performed with a contracted, high-frequency version of the prototype wavelet, while frequency analysis is performed with a dilated, low-frequency version of the same wavelet. Because the original signal or function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data operations can be performed using just the corresponding into a function in the frequency domain. The signal can then be analyzed for its frequency wavelet coefficients [3].

2. WAVELET ANALYSIS
Fourier transform gives the frequency information of the signal, but it doesn't tell us when in time these frequency components exist. The information provided by the integral corresponds to all time instances because the integration is done for all time intervals. It means that no matter where in time

the frequency f appears, it will affect the result of the integration equally. This is why Fourier transform is not suitable for non-stationary signals [4].

2.1. Short time fourier transform

To overcome the weakness described above, short time Fourier transform (STFT) was developed. In STFT the signal is divided into small segments which can be assumed to be stationary. In STFT of signal multiplied by a window function within the Fourier integral. If the window length is infinite, it becomes the FT. In order to obtain the stationary, the window length must be short enough. The narrower windows affords better time resolution and better stationary, but at the cost poorer frequency resolution. One problem with STFT is that one can't know what spectral components exist at what points of time. One can only know the time intervals in which certain band of frequencies exist [5].

2.2. Continuous wavelet transform

The continuous wavelet transform evolved as an alternative approach to STFT to overcome the resolution problem [2]. The wavelet transform is similar to STFT in that the signal is multiplied by a function similar to windows function in STFT, but the transform is done separately for different segments of the signal. The main differences between STFT and the CWT are that in CWT the width of the window is changed as the transform is computed for every signal spectral component. The CWT is defined by,

$$CWT(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(\tau)\varphi(\frac{\tau - b}{a})d\tau. \quad (1)$$

2.3. Discrete wavelet transform

The wavelet transform solves the dilemma of resolution to a certain extent. Multiresolution analysis (MRA) analyses the signal at different resolutions. MRA (also WT) is designed to give good time resolution or poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies. This approach makes sense especially when the signal at hand has high frequency components for short duration and low frequency components for long duration.

The Discrete Wavelet Transform (DWT) is easier to implement than Continuous Wavelet Transform (CWT) [1, 6, 7, 8]. The CWT was computed by changing the scale of the analysis window, shifting the window in time, multiplying by the signal, and integrating over all times. In the discrete case, filters of different cut off frequencies are used to analyze the signal at different scales. The signal is passed through a series of high pass filters to analyze the high frequency components and it is passed through a series of low pass filters to analyze low frequencies [9]. The wavelet decomposition results of a signal are called DWT coefficients.

3. WAVELET TRANSFORMS VERSUS FOURIER TRANSFORMS

The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space. Fourier sine and cosine functions are not. This localization feature, along with wavelets' localization of frequency, makes many functions and operators using wavelets "sparse" when transformed into the wavelet domain. This sparseness, in turn, makes wavelets useful for a number of applications such as data compression, feature detection in images, and noise removal from time series.

One way to see the time-frequency resolution differences between the two transforms is to look at the basis function coverage of the time-frequency plane. If the windowed Fourier transform is used, the window is simply a square wave. The square wave window truncates the sine or cosine function to fit a window of a particular width. Because a single window is used for all frequencies in the WFT, the resolution of the analysis is the same at all locations in the time-frequency plane.

An advantage of wavelet transforms is that the windows vary. In order to isolate signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basis functions. A way to achieve this is to have short high-frequency basis functions and long low-frequency ones. This happy medium is exactly what you get with wavelet transforms.

One thing to remember is that wavelet transforms do not have a single set of basis functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. Thus wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

4. CWT OF SINUSOIDAL SIGNALS APPLICATION

In the analysis investigated in this paper, the signal is composed of two sines with regard to the frequencies 50 and 150 Hz, respectively. Here maximum frequency is 150 Hz and since $F_s \geq 2F = 300$ Hz, sampling frequency is chosen as 1000 Hz. Also an additional noise is added to the signal. By using several wavelet functions (db1,db3 etc), approximation and detail coefficients of six levels are obtained and the effects of these factors on the signal are investigated. Also scaling factor and the gain of the noise effects are examined. If we apply wavelet transform to obtain the signals for each frequencies without noise according to approximation and detail coefficients, Figure 1 is obtained.

Figure 1 shows the obtained signals approximation coefficients at 6 levels. Here we loose additional noise after sixth level and at approximation coefficients from the first to fifth level we can see the effect of noise, easily. Here db3 function is used as wavelet function. Because by using db3, the most similar approximation coefficients to the signal are obtained for first level. According to the other db functions, the noise effect is canceled at earlier levels. For detail coefficients, at first level we can see the most effect of noise.

If we change the gain of the noise 10 times for investigating the effect of noise to signal, more noisy signal is obtained. From power spectrum density function, the noise effect can be seen at nearly all frequencies and when the scale factor of sinusoidal signal is changed by 4 with the 10 times gain of noise, Figure 2 is obtained.

When the noise effect and the scale factor is changed, the sinusoidals can be seen sixth level by db3 function, again. For investigating the effect of scale factor, Figure 3 is obtained.

Figure 3 shows that small scale coefficients correspond to high frequency mean for the signal noise and high scale coefficients correspond to low frequencies mean sinusoidals for this signal. For example, in Figure 3(b) for small scale coefficient (i.e 0.25), absolute coefficients are seen very frequently. And as the scale value increases, absolute coefficients become sparse and periodic signals are seen. This situation corresponds to used sinusoidal signals for this application.

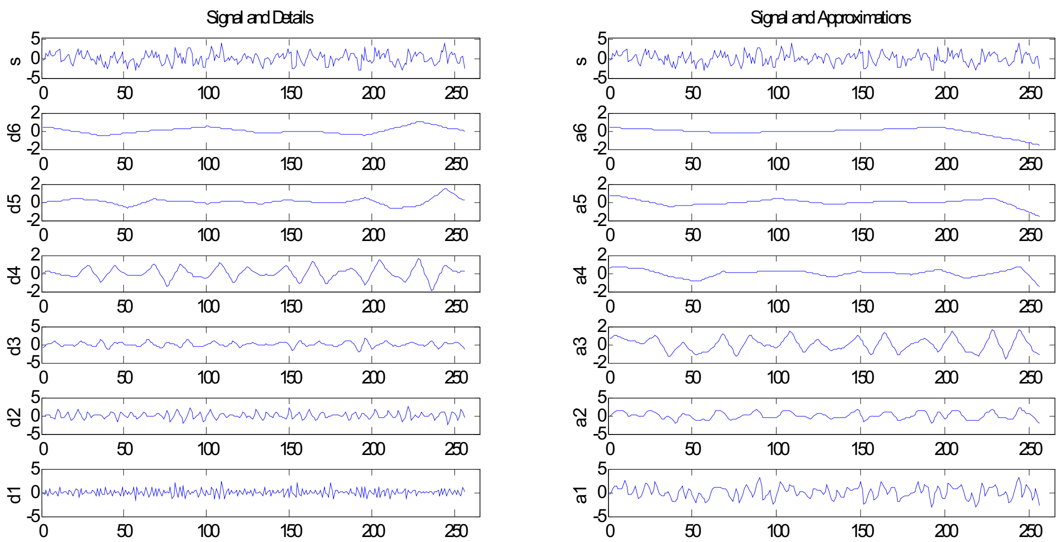

Figure 1. Approximation and detail coefficients obtained for 6 levels, respectively.

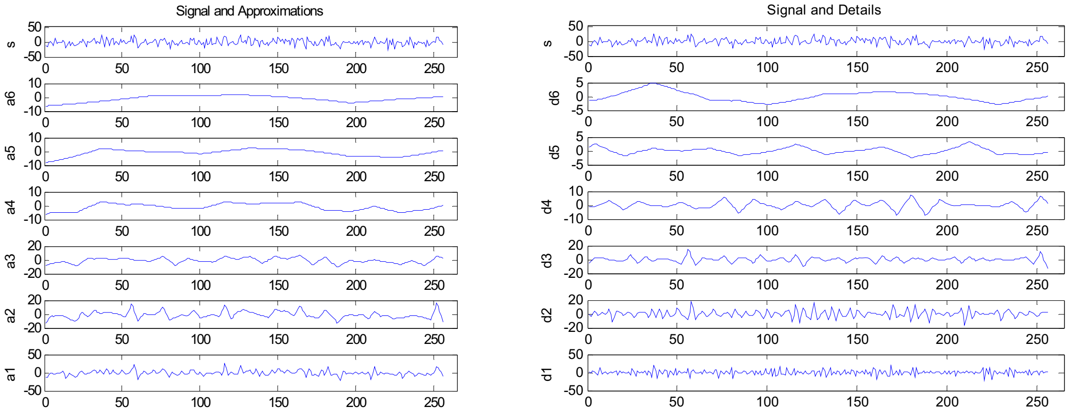

Figure 2. Approximation and detail coefficients when the scale factor of sinusoidal signal is changed by 4 with the 10 times gain of noise.

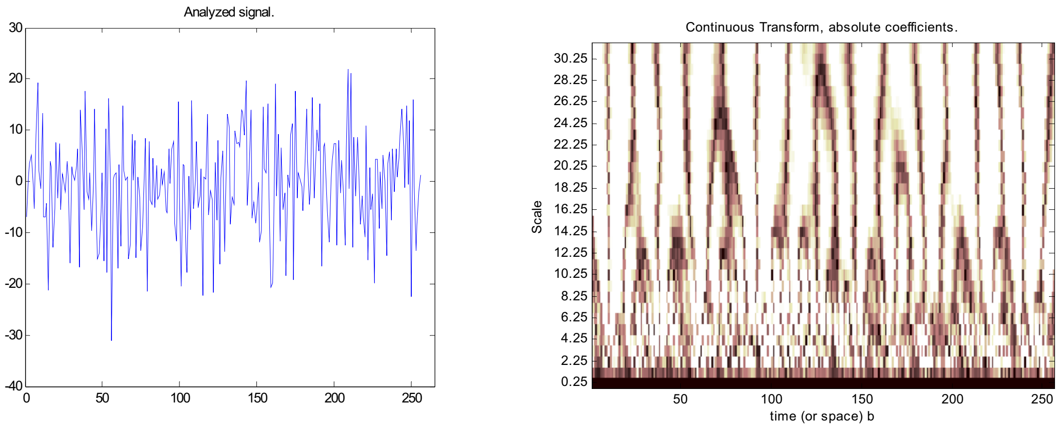

Figure 3. (a) Analyzed signal (b) Analyzed signal and continuous transform absolute coefficients.

5. CONCLUSIONS

Fourier transform shows what frequencies exist in the signal and tells how much of each frequency exists in a signal. Also the time and frequency information can not be seen at the same time.

STFT provides some information about both when and at what frequencies a signal event occurs. Precision is determined by the size of the window.

Wavelet analysis is a form of 'Multiresolution Analysis', which means that wavelet coefficients for a certain function contain both frequency and time domain information.

The continuous wavelet transform was developed as an alternative approach to the short time Fourier transform to overcome the resolution problem.

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