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NUMERICAL ANALYSIS OF THE ROTOR OF A HIGH-SPEED INTEGRATED FLYWHEEL ENERGY STORAGE SYSTEM

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ABSTRACT
This work investigates the mechanical behaviour of a rotor in working conditions. The rotor is a part of the high-speed integrated flywheel energy storage system designed at University of Berkley. It has a sinusoidal profile in order to achieve a sinusoidal MMF waveform with no harmonics. Mechanical behaviour is analysed using SolidWorks software. Firstly, the stress analysis of the rotor subjected to the centrifugal force is investigated with angular velocity of75000 rpm. Areas of maximum stresses are found and it is shown that the basic analytical solutions for maximum stresses in rotating discs cannot be used for this type of rotors. Thus, the numerical analysis seems to be the only option for this purpose. Secondly, the frequency analysis is carried out and main harmonics are identified.
Keywords: SolidWorks, FEM, stress and strain

1. INTRODUCTION
Rotating elements or rotors are used in machinery such as electric generators, heat or hydraulic machines (turbines, turbo pumps, centrifugal pumps, etc.), synchronous and asynchronous electric machines and many others [1]. For such elements, having high (rotational) speed, it is very important to evaluate stresses caused by centrifugal forces. In many cases the usage of simple analytical solutions, where rotating elements are simplified as discs of constant width and radius, suffice. However, there are cases where analytical solution cannot accurately predict behaviour of rotating elements and numerical analysis is necessary. This work investigates numerically a behaviour of a high-speed rotor, being a part of the high-speed integrated flywheel energy storage system designed at University of Berkley.

2. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL
In order to achieve sinusoidal waveform of magnetomotive force without harmonics (MMF) Tsao [2] used a sinusoidal flywheel for an electric motor used in aviation. He gave a detailed analysis in terms of electro-technical aspects, but stress and frequency analyses were omitted or only roughly investigated. This work is focused on detailed numerical stress and frequency analysis of this flywheel/rotor.
Figure 1 shows the sketch of the flywheel with three regions having different cross sections and the flywheel in reality. Two sinusoidal sections are designed in such a way that the gap between the stator and the rotor varies from 2 to 10 mm, thus producing a required shape of magnetomotive force. Maximum diameter of the flywheel is 112 mm and the length is 168.9 mm. More details about geometry can be found in [2].

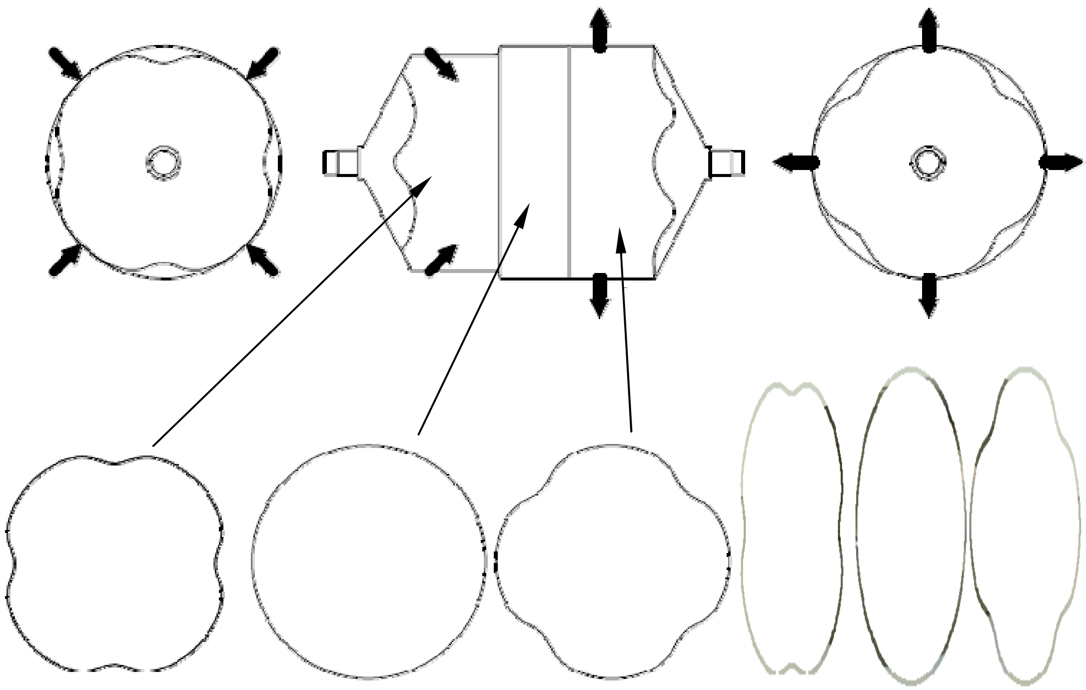

Figure 1. Sinusoidal flywheel with characteristic cross-sections [2]

In order to analyse the rotor numerically a 3D model is produced using SolidWorks software [3], as shown in Fig. 2-left. Due to double symmetry only a quarter of domain is modelled with total of 7431 elements (Fig.2-right; mesh is not shown). Three lines shown in the figure are used as entities to obtain stress distribution along them (in radial direction) and compare it with analytical predictions for circular rotating discs. Material is modelled as Aircraft Alloy Steel 300M, having modulus of elasticity of 205 GPa, Poisson's ration of 0.28, density of 7870 kg/m^3 and yield stress of 1580 MPa, which is generally used for highly-demanding aviation components.

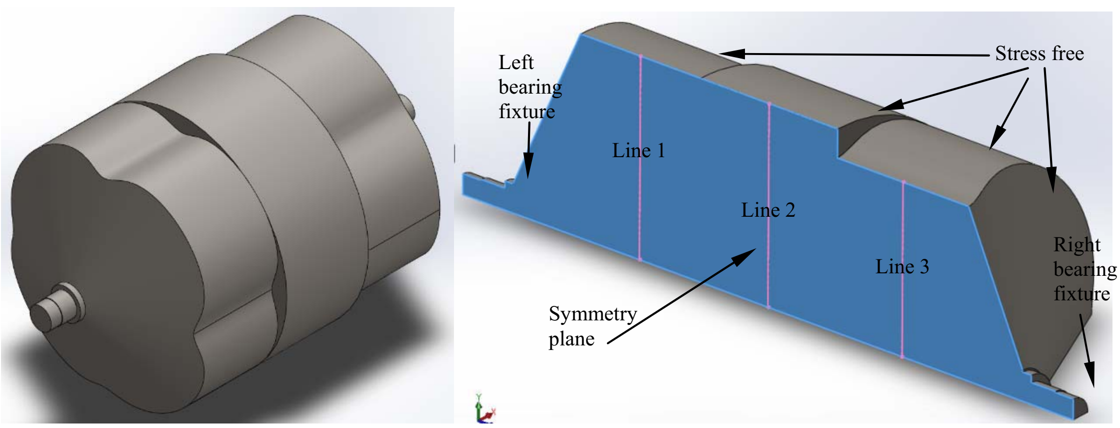

Figure 2. Sinusoidal rotor: left – 3D model; right – boundary conditions

Two cut surfaces are modelled as symmetry planes (only one is visible in Fig.2-right), whereas the outer surfaces of the three main sections are modelled as stress free, as indicated in the figure. The right bearing pin is modelled using fixed boundary condition in radial and axial direction (radial bearing; rotation is allowed), whereas the left bearing is modelled using fixed boundary condition in radial direction only (radial-axial bearing; rotation and axial movements are allowed). Centrifugal load/force, is taken into account by setting constant angular velocity of 75000 rpm. More detail can be found in [4].

In order to investigate the stresses in the rotor, stress distribution in three sections is compared to known analytical solutions for full circular disc of finite length. To this end, von Mises equivalent stress for triaxial stress state is used [1], ie.

$$\sigma_e = \sqrt{\sigma_r^2 + \sigma_h^2 + \sigma_z^2 - \sigma_r \cdot \sigma_h - \sigma_r \cdot \sigma_z - \sigma_h \cdot \sigma_z} \qquad \ldots(1)$$

where principal stresses are (radial, tangential and axial, respectively)

$$\begin{aligned} \sigma_r &= \frac{3+\nu}{8} \cdot \sigma_0 \cdot \left(1-\rho^2\right) \\ \sigma_t &= \frac{3+\nu}{8} \cdot \sigma_0 \cdot \left(1 - \frac{1+3\cdot\nu}{3+\nu} \cdot \rho^2\right) \\ \sigma_z &= \frac{\nu}{4\cdot(1-\nu)} \cdot \sigma_0 \cdot \left(1+\beta^2-2\rho^2\right) \end{aligned} \qquad \ldots(2)$$

ν is the Poisson's ratio (it has to be noted that Poisson's ratio in these calculations is replaced by $\frac{\nu}{1-\nu}$ due to finite width of the flywheel [1]), $\sigma_0=\gamma\cdot\omega^2\cdot R_e^2$ is the reference stress, $\rho=r/R$is non-dimensional variable, $\beta=R_i/R_e$ is non-dimensional constant, r is a distance from the axis, R_i and R_e are the inner and outer disc radius, respectively, γ is the specific weight and ω is the angular velocity.

3. RESULTS

Once the model is produced, stress analysis is carried out with the problem being treated as steady-state, ie. static analysis is applied. Figure 3 shows equivalent stress distribution in the model. It can be observed that the maximum stresses are in the regions with sinusoidal cross-sections, with the most critical regions being the troughs. The values in these regions are much higher than in the other parts and reach 630 MPa.

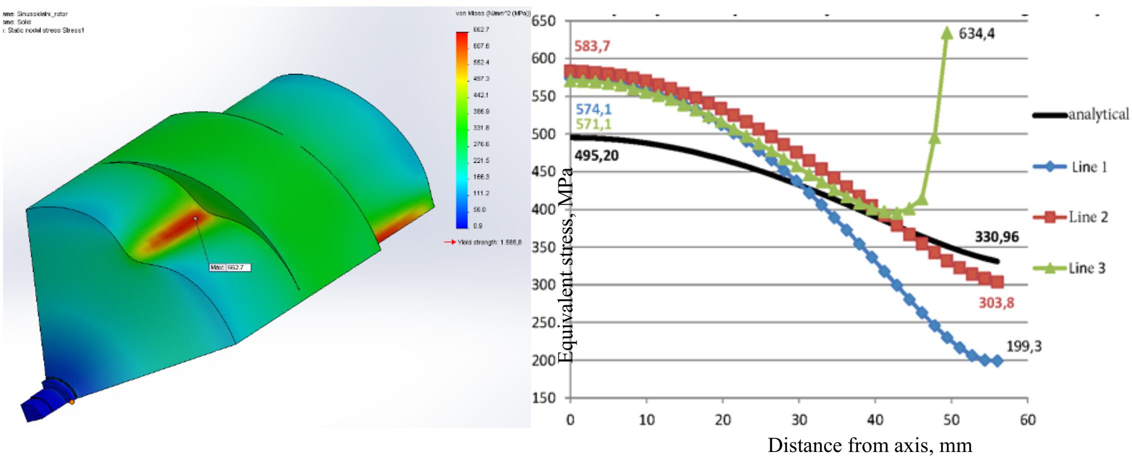

Figure 3. Equivalent stress distribution: left – spatial distribution, right – distribution along lines

By comparing stress distributions along characteristic lines defined in Fig. 2, it can be seen that stresses follow nicely the trend of analytical predictions, but with significant variations. In all cross sections the maximum stresses are along the axis and more than 20% higher than the analytical predictions. The only exceptions are the through regions of the sinusoidal cross-sections (outer surface), where stresses do not follow analytical trend and reach the peak of 634.4 MPa, which is higher than the maximum predicted stress in the axis. The main reason for this is the stress concentration caused by non-uniform cross-section and the vicinity of the middle part of the flywheel with sudden change of geometry.

In addition to the stress analysis, frequency analysis is also carried out, where natural frequencies of the flywheel are identified. The model is identical to the steady-state stress model, the only difference being the type of the analysis (the frequency analysis instead of the static one). Therefore, it is not necessary to make any further modifications to the model, but only run different type of analysis.

As a result of analysis, first five natural frequencies of the system are found. Following values are obtained, from the first to the fifth mode: 2776 Hz, 18245 Hz, 21568 Hz, 21843 Hz and 25182 Hz. Figure 4 shows amplitude distributions for the first two natural frequencies. It is interesting to note that the first natural frequency falls into range of 75000 rpm, ie. forced frequency will meet natural frequency during flywheel acceleration and resonance will occur.

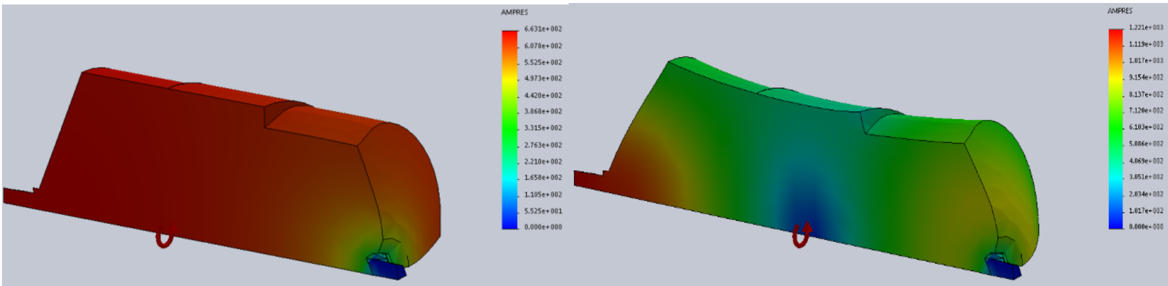

Figure 4. Frequency analysis of the flywheel – amplitude distribution: left – 1st natural frequency, right – 2nd natural frequency

4. CONCLUSION

A sinusoidal flywheel, acting as a rotor of an electric motor as well, is analysed in terms of stresses and natural frequencies. To this end, SolidWorks Simulation software is used; namely, static and frequency analyses are applied. It is shown that analytical predictions for a simple disc of constant radius cannot be used when estimating maximum stresses. Unlike stresses for circular discs, with maximum stresses along the axis, maximum stresses for sinusoidal discs are on outer surfaces, where stress concentration causes higher stresses. However, the stresses are still far beyond yield stress (around 3 times lower) meaning that the rotor can be used at even higher speeds, exceeding 100000 rmp. Frequency analysis shows that the first natural frequency is below rotational frequency, meaning the resonance occurs and one has to take care when designing these systems. When increasing angular velocity above 120000 rpm, other frequency modes would be reached as well.

It has to be also noted that SolidWorks Simulation software proved to be a great tool for a quick stress and frequency analysis, which is very important in conceptual design phase. It required minimum of effort to produce model and prepare it for both analyses. The problem is to be further investigated by considering it as a dynamic one, ie. simulation is to be carried out from the resting point to the point of reaching the working speed.

5. REFERENCES

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