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NEWSVENDOR EXTENSIONS WITH BACKORDER AND EMERGENCY SUPPLY OPTION

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ABSTRACT
In this paper new extension of newsvendor model is developed. This extension allows modeling of the situation with the percentage of customers willing to wait for the next regular delivery and with the percentage of customers not willing to wait for next regular delivery but for emergency delivery. The developed model works on the principle of activation backorder option and emergency supplies option from the moment when stock depletes.
Keywords: inventory, extensions newsvendor model, newsvendor model with backorder and emergency supply option

1. INTRODUCTION
There are a number of logistic systems and cases in which orders may be placed only at certain times. Ordering dates of these systems are known in advance, because they are imposed by some limitations of the logistic system itself, or are due to the specific characteristics of the products kept on stock. Objective of inventory control, in these systems, is to determine the number of orders that will be sufficient to cover demand between two consecutive orders. Ordering too little, in these systems and cases, means that the demand cannot be satisfied, and additional costs will occur, such as additional order costs, lost sales costs, reputation and business credibility loss, future contracts and sales loss, etc. Ordering too much, means the occurrence of excessive inventory levels that generate costs whose values depends on the nature of the product. Modeling of described inventory control problems can be successfully done using newsvendor model and its extensions. Product nature and logistic system limitations determine what newsvendor extension is adequate. Optimal newsvendor extension is the one derived with respect to all assumptions of the real system, and the goal is to achieve the same service level with smaller order quantity.

2. LITERATURE REVIEW
Newsvendor problem has been present in the literature for over 100 years [1], and is the one of the most famous models in the operations management and operations research, in general [2]. Newsvendor model still attracts the attention of a large number of authors in recent years [3]. Since its introduction, applicability of newsvendor model is more and more intensive, which has resulted in numerous extensions and articles.

Applicability of newsvendor model is manifold. Inventories in the food and the clothing industry are often modeled using newsvendor model [4]. Newsvendor model is also used in modeling and solving problems in the production capacity management, and in the service industries, such as airline and hotel reservations [5, 6]. As the lifetime of the product continues to shorten, the importance of the newsvendor models grows, so many newsvendor model extensions are developed in the last few years [3, 7, 8].

For derivation of newsvendor extension with backorder and emergency delivery options, and also for the definition of the classic newsvendor model and other extensions mentioned in this paper, following notation has been used: x – demand, random variable; $f(x)$ – probability density function of x; $F(x)$ – cumulative distribution function of x; P – selling price per unit; C – buying price per unit; C_1 – buying price per unit in case of emergency delivery; V_1 – salvage value per unit; S – shortage penalty cost per unit; b – percent of demand that can be satisfied from the next regular order, in case of backorder; t – percent of demand that can be satisfied from the emergency delivery; Q – ordering quantity, decision variable.

Optimal ordering quantity for the classic newsvendor model is obtained by maximizing expected profit, see [3,9]:

$$Q^* = F^{-1}\left(\frac{P+S-C}{P+S-V}\right). \qquad \ldots(1)$$

Identical equation for the optimal ordering quantity can be obtained by minimization of expected shortage and excess inventory costs [3, 10].

Extension of the newsvendor model with percent of customers not willing to wait for the next regular order to satisfy demand, but for emergency delivery, is developed in [11]:

$$Q_H^* = F^{-1}\left[\frac{(1-t)(P+S)-C+C_1 t}{(1-t)(P+S)-V+C_1 t}\right] \qquad \ldots(2)$$

Extension of the newsvendor model with percent of customers willing to wait for the next regular order to satisfy demand, is developed in [12]:

$$Q_B^* = F^{-1}\left(\frac{(1-b)(P+S-C)}{(1-b)(P+S)+Cb-V_1}\right) \qquad \ldots(3)$$

3. DERIVATION OF THE NEWSVENDOR EXTENSION WITH BACKORDER AND EMERGENCY DELIVERY OPTION

Extension developed in this paper represents sublimation of models defined by (2) and (3).

Profit in one planning period is:

$$\pi = \begin{cases} (P-C)Q + (P-C)(x-Q)b + \\ +(P-C_1)(x-Q)t - (1-t-b)S(x-Q), & \text{if } x \geq Q \\ (P-C)x - (C-V)(Q-x), & \text{if } x < Q \end{cases}, \qquad \ldots(4)$$

where: $(P-C)Q$ – profit of Q units sold if $x \geq Q$; $(P-C)(x-Q)b$ – profit of $(x-Q)b$ units sold if $x \geq Q$; $(P-C_1)(x-Q)t$ – profit of $(x-Q)t$ units sold if $x \geq Q$; $(1-t-b)S(x-Q)$ – total penalty (shortage) cost if $x \geq Q$, $(P-C)x$ – profit of x units sold if $x < Q$; $(C-V)(Q-x)$ – total cost of excess inventory if $x < Q$.

Expected profit in one planning period is:

$$E(\pi) = [P - C - (P-C)b - (P-C_1)t + (1-t-b)S] \int_Q^{+\infty} Qf(x)dx +$$

$$+[(P-C)b+(P-C_1)t-(1-t-b)S]\int_Q^{+\infty} xf(x)dx+$$
$$+(P-V)\int_0^Q xf(x)dx-(C-V)\int_0^Q Qf(x)dx. \qquad \text{...(5)}$$

First derivative of function of expected profit can be obtained using Leibnitz's rule. First derivative of members of the function of expected profit are given by expressions from (6) to (9):

$$\frac{\partial \int_Q^{+\infty} Qf(x)dx}{\partial Q}=\int_Q^{+\infty} f(x)dx-Qf(Q), \qquad \text{...(6)}$$
$$\frac{\partial \int_Q^{+\infty} xf(x)dx}{\partial Q}=-Qf(Q), \qquad \text{...(7)}$$
$$\frac{\partial \int_0^Q xf(x)dx}{\partial Q}=Qf(Q), \qquad \text{...(8)}$$
$$\frac{\partial \int_0^Q Qf(x)dx}{\partial Q}=\int_0^Q f(x)dx+Qf(Q)\,. \qquad \text{...(9)}$$

After replacing and ordering derivatives defined by (6) to (9), we get first derivative of the function of expected profit over Q:

$$\frac{\partial E(\pi)}{\partial Q}=(P-C+S)[1-F(Q)]-(P-C+S)b[1-F(Q)]-$$
$$-(P-C_1+S)t[1-F(Q)]-(C-V)F(Q). \qquad \text{...(10)}$$

After the first derivative of the function of expected profit is set to zero, we get:

$$F(Q_{HB}^*)=\frac{(1-t-b)(P+S)-(1-b)C+C_1t}{(1-t-b)(P+S)+Cb-V+C_1t}, \qquad \text{...(11)}$$

where Q_{HB}^* is optimal ordering quantity, under condition that the function of expected profit is concave, which can be proved by calculating its second derivative using Leibnitz's rule.

$$\frac{\partial^2 E(\pi)}{\partial^2 Q}=f(Q)[(P+S)(b+t-1)-Cb-C_1t+V]. \qquad \text{...(12)}$$

From expression (12) one can see that for every value of Q, value of the function of expected profit is less than zero, so we conclude that the function of expected profit has maximum and it is in Q_{HB}^*. Explicit expression for calculating optimal ordering quantity Q_{HB}^* follows from (11):

$$Q_{HB}^*=F^{-1}\left(\frac{(1-t-b)(P+S)-(1-b)C+C_1t}{(1-t-b)(P+S)+Cb-V+C_1t}\right). \qquad \text{...(13)}$$

If we compare expression for optimal ordering quantity of extension developed in this paper (13) with expressions for optimal ordering quantity of the classical newsvendor model (1), or newsvendor extension with emergency delivery (2), or newsvendor extension with backorder option (3) it can be proved that $Q_{HB}^*\le Q^*$, $Q_{HB}^*\le Q_H^*$, $Q_{HB}^*\le Q_B^*$, under condition that $V<C<C_1<P+S$.

4. CONCLUSION

Developed extension of newsvendor model with backorder and emergency order options allows achieving the same service level with lower or at least the same inventory level. In situations when there are percentage of customers who are willing to wait for the next regular delivery and when there are percentage of customers not willing to wait for the next regular delivery but for emergency delivery, developed extension will always achieve the same service level with lower inventory levels compared to classic newsvendor model, newsvendor model with emergency delivery and newsvendor model with backorder option.

Developed extension enables modeling of the situations when there are no customers willing to wait for the next regular and/or emergency delivery. If there are no percentage of customers who are willing to wait for the next regular delivery ($b = 0$) and next emergency delivery ($t = 0$), then developed extension becomes classic newsvendor model. In situations when there are customers who are willing to wait for the next regular order, newsvendor model with backorder option, that is triggered after the stock out, is more efficient than the classic newsvendor model, and the newsvendor model with backorder option, that is triggered at the beginning of the planning period, is more efficient than both models. In situations where all customers are willing to wait for the next regular order, both extensions of the newsvendor model developed in this paper, will give the same result. The advantage of the developed extension compared to classic newsvendor model, newsvendor models with emergency delivery option and newsvendor model with backorder option is its versatility and efficiency in all situations.

Developed extension could be even more efficient if it can enable activation of the emergency delivery and backorder options from the very beginning of the planning period, and not from the time when stock depletes.

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