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NUMERICAL METHOD FOR THE KINEMATIC ANALYSIS OF THE MCPHERSON GUIDING MECHANISMS USED FOR THE REAR WHEELS OF THE PASSENGER CARS

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ABSTRACT
The paper deals with a numerical method for the kinematic analysis of the McPherson guiding mechanisms of the vehicle rear wheels with independent suspension. The kinematics of the mechanism is modeled by a non-linear system of 8 equations, regardless of the complexity of the mechanism in terms of number of elements, which is then solved by using the Newton-Kantorovich approach. A computer program was developed and tested on various McPherson suspension configurations.
Keywords: passenger car, wheel guiding mechanism, McPherson suspension, kinematic analysis.

1. INTRODUCTION
The independent guidance of the rear wheels of the passenger cars is achieved by means of a suspension mechanism for each wheel. Compared to the guidance of the beam axle, the following advantages are ensured through wheel independent guidance: less allocated space, reduced weight, there is no mutual influence of the wheels, increased comfort. For the nowadays cars, the multi-link suspension configurations are frequently used. The McPherson design is a special type of suspension, at which the shock absorbers are integrated in the structural & kinematic model (with influence on the kinematic behavior), unlike the conventional multi-link mechanisms where the shock absorbers only play the role of damping elements (thus being neglected in the kinematic analysis) [1].
The kinematic analysis of the wheel guiding mechanisms can be done by using automated formalisms, such as incorporated into the commercial MBS (Multi-Body Systems) software environment (e.g. ADAMS of MSC Software), which automatically formulate and solve the motion equation systems in accordance with the specific types of constraints between bodies [2, 3], or classical methods, which consist of the analytical modeling of the kinematic behavior, usually by geometrical or vectorial methods [4, 5, 6]. Having in view the reduced complexity of the kinematic model (as result of the specific simplifying assumptions), the study can be approached by classical methods, with the benefit of customizing the computing programs.

2. KINEMATIC ANALYSIS ALGORITHM
The general form of the rear wheel suspension mechanism is that corresponding to the spatial guidance of a body on five fixed spheres having the centers on car body (fig. 1). The degree of mobility of the mechanism can be computed in accordance with the number of joints mobilities (Σm_i), the motion space (s), and the number of independent kinematic loops (k), as follows:

$$DOM = \Sigma m_i - S \cdot k = 30 - 6 \cdot 4 = 6 = 1 + 5, \tag{1}$$

the five rotations of the guiding links around their own axes being passive from kinematic point of view. Therefore, the general multi-link wheel guiding mechanism has one active degree of mobility, which corresponds to the vertical travel (up-down) of the wheel.

By structural and constructive customizations of the general multi-link mechanism shown in figure 1, several variants of wheel guiding mechanisms can be defined, one of the most used mechanism being the McPherson design (fig. 2), which is obtained by replacing the set on upper links (3-3') and their corresponding spherical joints with a tetra-mobile joint, thus materializing a cylinder (2 - common part with the wheel carrier) & piston (3) assembly (i.e. the shock absorber / damper).

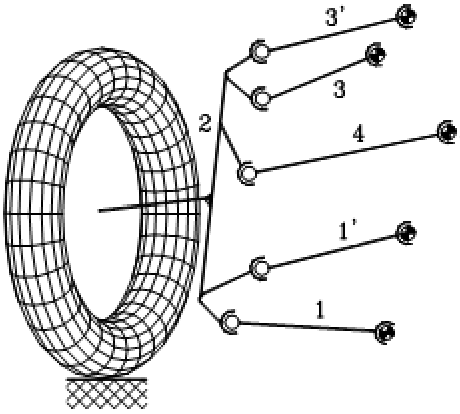

Figure 1. Multi-link wheel guiding mechanism.

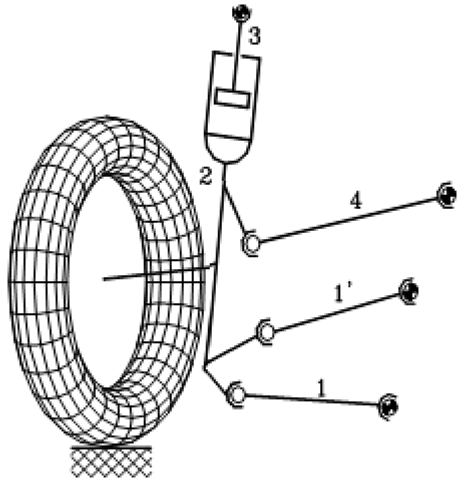

Figure 2. McPherson guiding mechanism.

The geometric model of the McPherson wheel guiding mechanism is defined by the following parameters (fig. 3):

- the coordinates of the points M_{0i} (i=1...3) and N_0 in which the guiding bars are connected to car body, in the global reference frame OXYZ attached to car body: X_{M0i}, Y_{M0i}, Z_{M0i}; X_{N0}, Y_{N0}, Z_{N0};
- the coordinates of the spherical joints M_i (i=1...3) by which the guiding bars are connected to wheel carrier, in the local reference frame $PX_PY_PZ_P$ attached to wheel carrier: $X_{Mi(P)}$, $Y_{Mi(P)}$, $Z_{Mi(P)}$;
- the orientation angles of the damper axis (Δ) in $PX_PY_PZ_P$: $\varepsilon_{\Delta(P)}$, $\lambda_{\Delta(P)}$;
- the static (initial) position of the wheel carrier, in OXYZ: X_P^0, Y_P^0, Z_P^0.

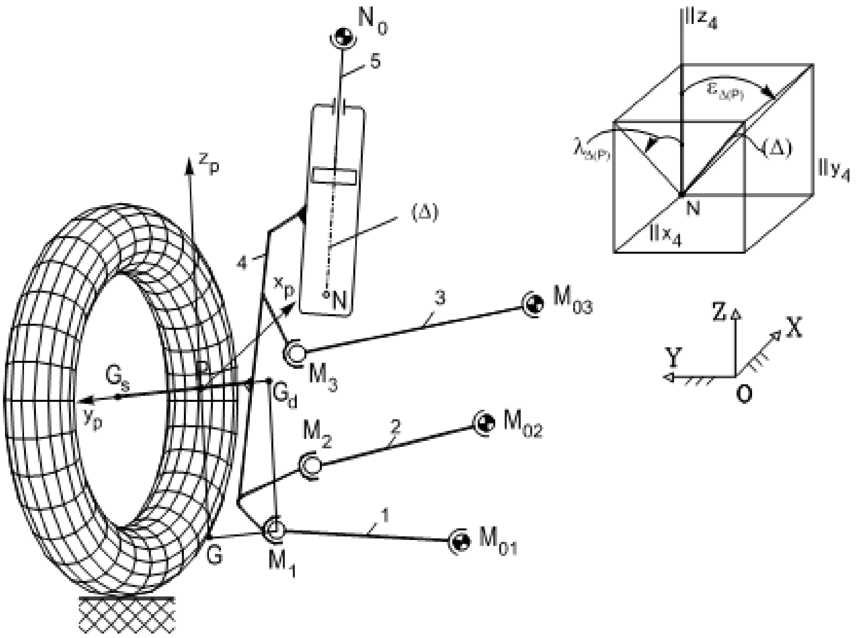

Figure 3. The geometric model of the McPherson guiding mechanism.

The global reference frame OXYZ has the axes parallel with the longitudinal (X), transversal (Y) and vertical (Z) technological axes of the vehicle, while the wheel reference frame $PX_PY_PZ_P$ has the origin P in the centre of the spindle axis Y_P.

The spatial motion of the wheel carrier is completely determined by its three non-collinear points, here called characteristic points. By the proposed method, the three points are the wheel center G_s and the projections G_d - G of the lower guiding point M_1 on the local axes Y_P - Z_P. The kinematic modeling of the mechanism is based on the rigid body conditions for wheel carrier (which are expressed by constant distances between the three characteristic points, F_1-F_3), as well as the constraint equations (the points through which the wheel carrier is guided in the relative motion to car body to describe specific geometric shapes, F_4-F_8), as follows:

$$
\begin{gathered}
F_1 = (X_G - X_{Gd})^2 + (Y_G - Y_{Gd})^2 + (Z_G - Z_{Gd})^2 - GG_d^2 = 0, \\
F_2 = (X_G - X_{Gs})^2 + (Y_G - Y_{Gs})^2 + (Z_G - Z_{Gs})^2 - GG_s^2 = 0, \\
F_3 = (X_{Gd} - X_{Gs})^2 + (Y_{Gd} - Y_{Gs})^2 + (Z_{Gd} - Z_{Gs})^2 - G_dG_s^2 = 0, \\
F_4 = (X_{M1} - X_{M01})^2 + (Y_{M1} - Y_{M01})^2 + (Z_{M1} - Z_{M01})^2 - l_1^2 = 0, \\
F_5 = (X_{M2} - X_{M02})^2 + (Y_{M2} - Y_{M02})^2 + (Z_{M2} - Z_{M02})^2 - l_2^2 = 0, \\
F_6 = (X_{M3} - X_{M03})^2 + (Y_{M3} - Y_{M03})^2 + (Z_{M3} - Z_{M03})^2 - l_3^2 = 0, \\
F_7 = (X_{N0} - X_N)\cdot Y_\Delta - (Y_{N0} - Y_N)\cdot X_\Delta = 0, \\
F_8 = (Y_{N0} - Y_N)\cdot Z_\Delta - (Z_{N0} - Z_N)\cdot Y_\Delta = 0,
\end{gathered}
\tag{2}
$$

where X_Δ, Y_Δ, Z_Δ are the global components (in OXYZ) of the damper axis vector,

$$
\begin{bmatrix} X_\Delta \\ Y_\Delta \\ Z_\Delta \end{bmatrix} = M_{40} \cdot \begin{bmatrix} \cos\varepsilon'_{\Delta(P)} \cdot \sin\lambda_{\Delta(P)} \\ \sin\varepsilon'_{\Delta(P)} \\ \cos\varepsilon'_{\Delta(P)} \cdot \cos\lambda_{\Delta(P)} \end{bmatrix}, \tag{3}
$$

M_{40} being the matrix that defines the orientation of the wheel carrier frame relative to OXYZ.
The unknowns that will be determined through the kinematic analysis are the global coordinates (in OXYZ) of the characteristic points, excepting the vertical coordinate of the wheel center Z_{Gs} which is the independent kinematic parameter, in accordance the number of degrees of mobilities of the mechanism, defined by equation (1). In the first three equations (F_1-F_3), the unknowns appear explicitly, while the following five equations (F_4-F_8) they appear as being implicit through the global coordinates of the guiding points:

$$
\begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} + \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \cdot \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix}_P, \tag{4}
$$

where M can be any of the guiding points M_1, M_2, M_3, or N, m_{ij} (i, j=1...3) are the components of the connection matrix M_{40} (the director cosines), while the coordinates of the center P are determined from the intersection of the spindle axis G_sG_d with the plane normal to G_sG_d taken through G.
The non-linear system (2) is solved by using the Newton-Kantorovici approach. The solution starts from the neutral position of the mechanism (vehicle in rest), for which the initial solution of the system can be can be accurately established. The solving involves the following steps: (a) setting the initial solution; (b) computing the coordinates of the wheel carrier center and of the wheel guiding points; (c) establishing the Jacobian of the system by derivating the functions F_1-F_8; (d) determining the new solution (in the first iteration), by using the Gauss-Jordan method; (e) testing the obtained error (the differences between the values of the unknown parameters in the first '1' and initial '0' iterations) - if the errors are satisfactory, then '1' is retained as new solution, otherwise the iterative process resumes from step (b), assuming as the initial solution in the new iteration the values of the unknowns from the previous iteration. The iterative process is finished when the difference between the values of the eight unknowns in two successive iterations 'm-1' şi ' m' reaches the imposed accuracy, the final solution being $\{X_G, Y_G, Z_G, X_{Gs}, Y_{Gs}, X_{Gd}, Y_{Gd}, Z_{Gd}\}_m$. For a current position of the wheel guiding mechanism, the non-linear system is solved in similar way, considering as initial solution the known prior position of the mechanism.
Thus, the measures that define the kinematic behavior of the mechanism can be defined, as follows (fig. 4):

- the orientation angles of the spindle axis, and their variations: toe angle δ (the symmetric angle that each wheel makes with the longitudinal axis of the vehicle), and camber angle γ (the angle between the vertical axis of the wheels and the corresponding axis of the vehicle):

$$\delta = \mathrm{arctg}\frac{X_{Gd} - X_{Gs}}{Y_{Gd} - Y_{Gs}}, \Delta\delta = \delta - \delta_0,$$
$$\gamma = \mathrm{arctg}\frac{Z_{Gd} - Z_{Gs}}{Y_{Gd} - Y_{Gs}}, \Delta\gamma = \gamma - \gamma_0; \quad (5)$$

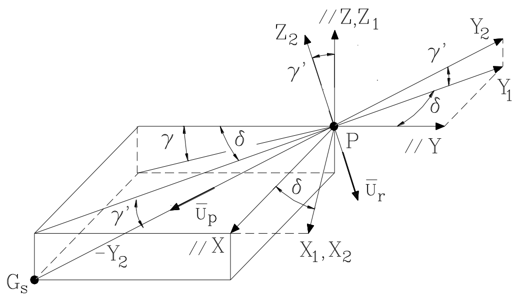

Figure 4. The spindle axis orientation angles.

- the wheel track (ΔE) and wheelbase (ΔL) variations - the displacements of the contact point K between wheel and road in transversal (Y) and longitudinal (X) directions:

$$\Delta E = Y_K - (Y_K)_0, \Delta L = X_K - (X_K)_0, \quad (6)$$

where the coordinates of the contact point K are obtained from the intersection of the vertical plane taht contains the spindle axis with the plane perpendicular on the spindle axis that passes through G_s, and the sphere with center in G_s and radius G_sK.

3. RESULTS AND CONCLUSIONS

Based on the above presented method, a computer program was developed by using the programming language C++. The program was tested on various types of guiding mechanism and kinematic regimes. For example, figure 5 shows the variations of the toe and camber angles when the wheel passes over a sinusoidal bump with the amplitude of 80 mm ($Z_{Gs} \in [-80, 80]$ mm). By comparison with other existing methods, the algorithm presented above brings important advantages, such as the large applicability field, the fast convergence of the non-linear systems, the accurate selection of the initial solution. For a further work, the author intends to integrate the method in a more complex algorithm for the kinetostatic analysis of the McPherson suspension.

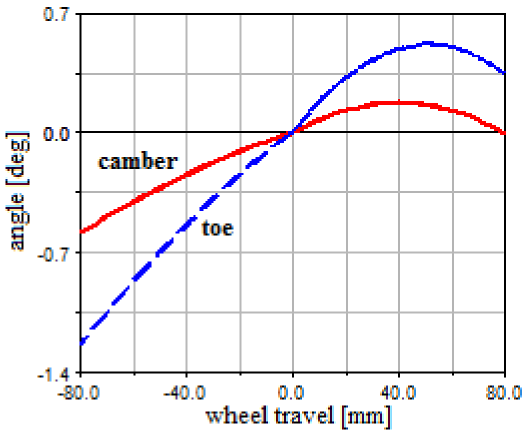

Figure 5. Toe & camber angle variations.

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